Machine Learning

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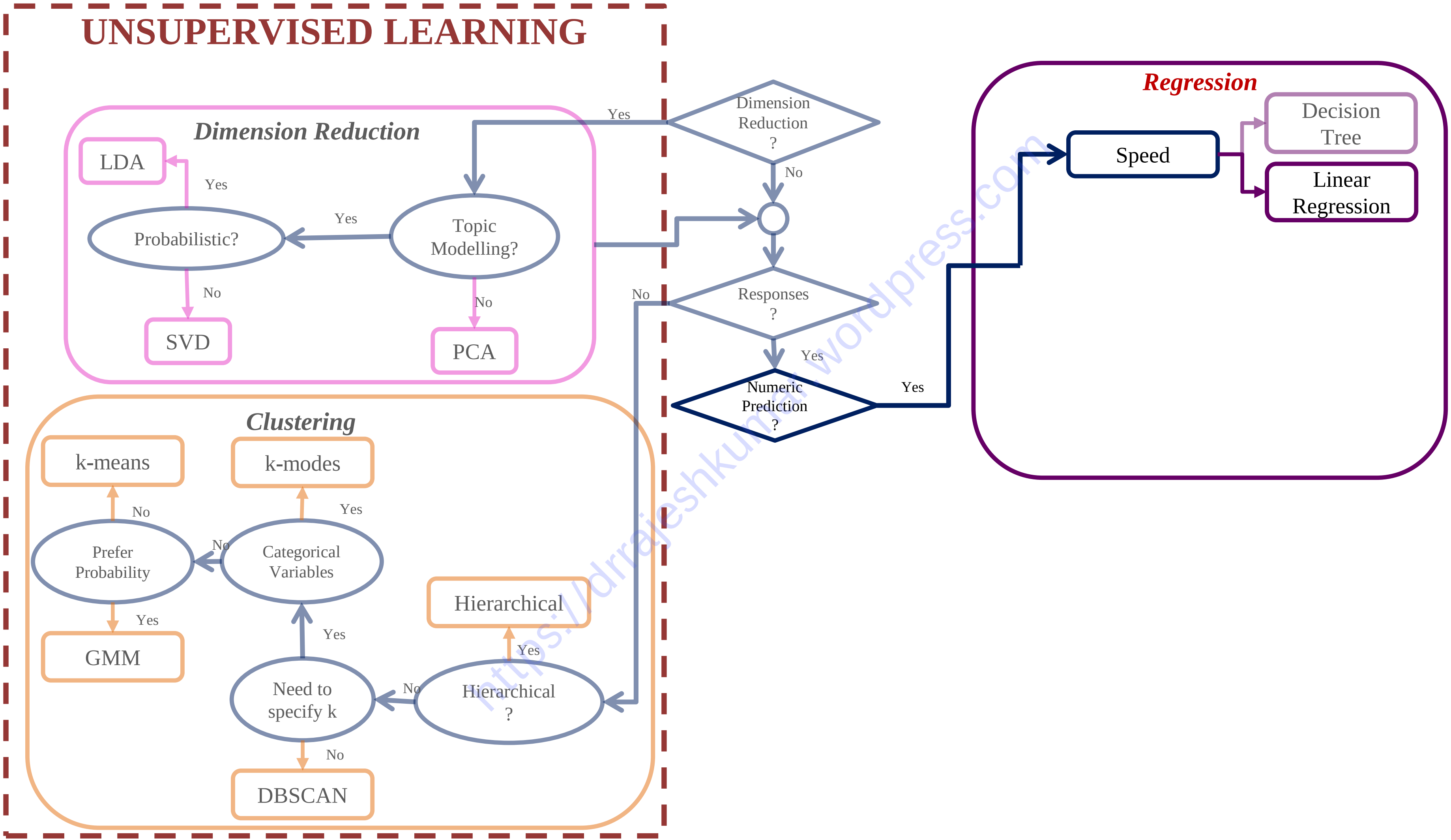


UNSUPERVISED LEARNING
Dimension Reduction
LDA
Yes
Probabilistic?
Yes
Topic Modelling?
No
SVD
No
PCA
Clustering
k-means
k-modes
No
Yes
Prefer Probability
No
Categorical Variables
Yes
GMM
Yes
Hierarchical
Yes
Need to specify k
No
Hierarchical ?
No
DBSCAN
Yes
Dimension Reduction ?
No
No
Responses ?
Yes
Numeric Prediction ?
Yes
Regression
Speed
Decision Tree
Linear Regression

Linear Regression

- Model linear relation of dependent variables with one or more independent variable
  - Simple Linear Regression (one independent variable)
  - Multiple Linear Regression (multi independent variable)
- Most applications fall into one of the following two broad categories:
  - Prediction, forecasting, error reduction
    Fit predictive model and than use for Prediction
  - Variation in the response variable
    Quality of strength between the responses

Linear Regression

- A linear regression model assumes that the linear relationship between the dependent variable *y* and the *p*-vector of regressors **x** (independent variable)

$$y_i = \beta_0 1 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + \varepsilon_i = \mathbf{x}_\mathbf{i}^\mathbf{T} \boldsymbol{\beta} + \varepsilon_i \qquad i = 1, \ldots, n$$

  - y_i is dependent variable for observation *i*
  - x_i is independent variable for observation *i*
  - ε_i is error term for the observation *i*
  - β_0 is intercept coefficient
  - β_1 is regression coefficient for independent variable

Linear Regression

- *n* equations are stacked together and written in matrix notation as

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{np} \end{pmatrix}}_{\mathbf{X}} \underbrace{\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}}_{\boldsymbol{\beta}} + \underbrace{\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}}_{\boldsymbol{\varepsilon}}$$

Linear Regression

- Estimated Model $\hat{\mathbf{y}} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ for given $\boldsymbol{\beta}$
- Objective of the regression model is to find the $\boldsymbol{\beta}$ to have the least possible error of estimated model $\hat{\mathbf{y}}$ from the data $\mathbf{y}$
- Error **e** used for fitting

$$\mathbf{e} = \hat{\mathbf{y}} - \mathbf{y}$$

Some Approaches	Objective Function
Least Square	$\mathbf{e}^T\mathbf{e}$
Ridge Regression	$\left(\sum_{i=1}^{n} e_i^2\right)^{1/2} = \left(\mathbf{e}^T\mathbf{e}\right)^{1/2}$
Lasso (least absolute shrinkage and selection operator)	$\left(\sum_{i=1}^{n} \lvert e_i \rvert\right)$

Linear Regression(least square)

- **Simple Linear Regression** model fitting with least square

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

- Objective is

$$\min_{\beta_0, \beta_1} \sum_{i=1}^{n} [y_i - \beta_0 - \beta_1 x_i]^2$$

- The solution can be obtain

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2} ; \hat{\beta}_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2}$$

Linear Regression(least square)

- **Multiple Linear Regression** model fitting with least square

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

- Objective is

$$= \min_{\boldsymbol{\beta}} \sum_{i=1}^{n} \left[y_i - \hat{y}_i \right]^2$$

$$= \min_{\boldsymbol{\beta}} (\mathbf{y} - \hat{\mathbf{y}})^{\mathrm{T}} (\mathbf{y} - \hat{\mathbf{y}}) = \min_{\boldsymbol{\beta}} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^{\mathrm{T}} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})$$

- The solution can be obtain

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^{\mathrm{T}} \mathbf{X} \right)^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{y}$$

Note: $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ should be non singular / invertible matrix

Linear Regression(least square)

- Correlation coefficient *r*

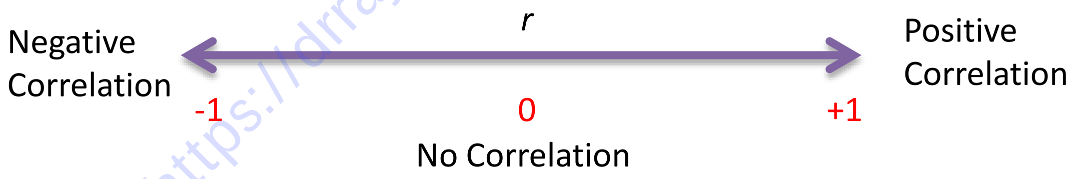
$$s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}}$$

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

Linear Regression(least square)

- Correlation coefficient *r*
  - Positive Correlation
    Direct relationship with independent variable
  - Negative Correlation
    Inverse relationship with independent variable
  - Correlation relationship strength is proportional to magnitude of *r*

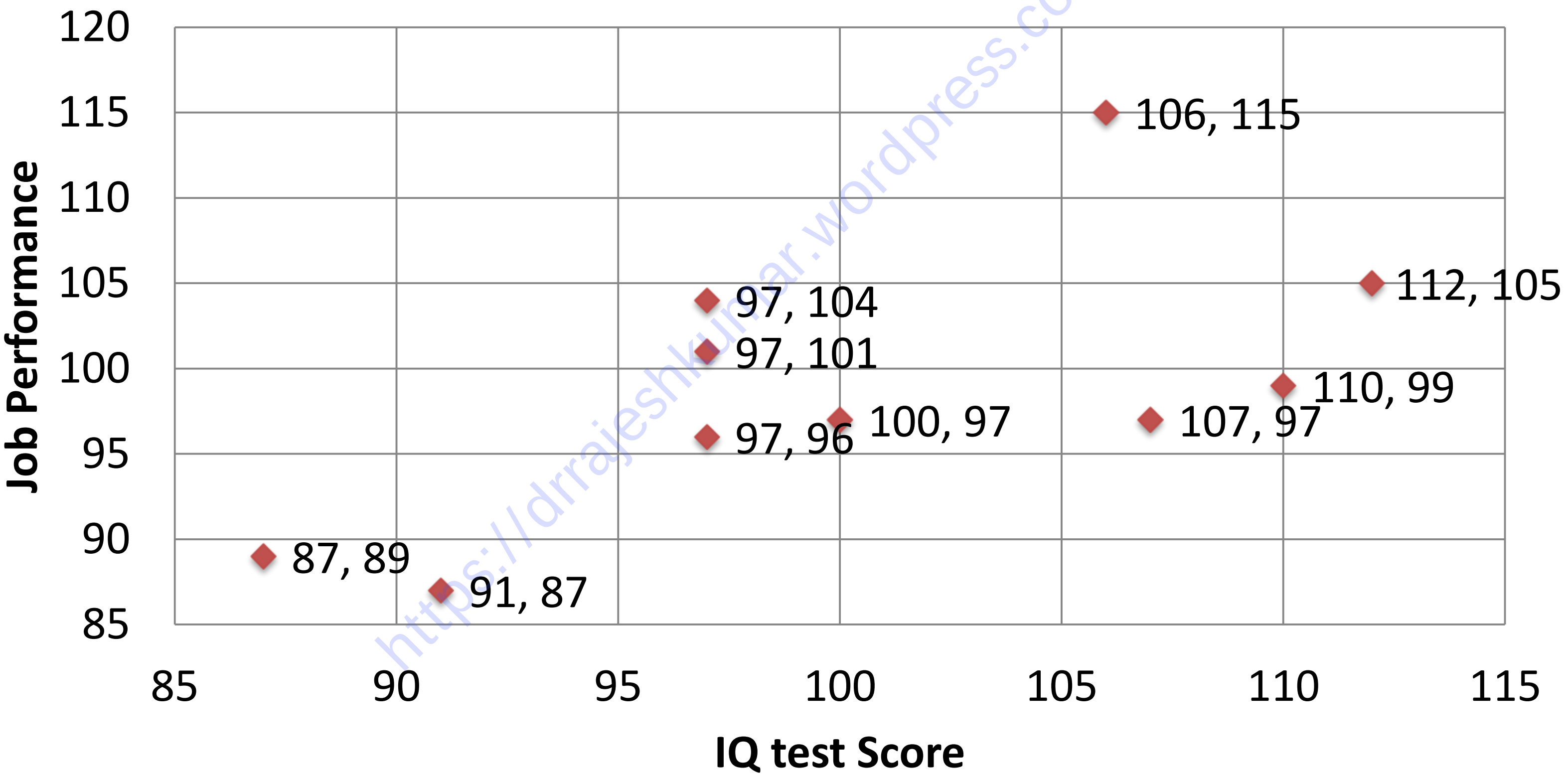


Example of Linear Regression

- Objective is to predict Job Performance from IQ
- Data comprising of 10 person information

Id	IQ	Performance
1	106	115
2	97	104
3	107	97
4	97	101
5	110	99
6	112	105
7	97	96
8	87	89
9	100	97
10	91	87

Example of Linear Regression



Example of Linear Regression

- Coefficients for linear regression

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{990 \times 101406 - 1004 \times 99776}{10 \times 101406 - (1004)^2} = 35.8762$$

$$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{10 \times 99776 - 1004 \times 990}{10 \times 101406 - (1004)^2} = 0.6287$$

- Simple linear regression model

$$\hat{y} = 35.8762 + 0.6287x$$

Example of Linear Regression