Termite Spatial Correlation Based PSO (TSC-PSO)

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Outline

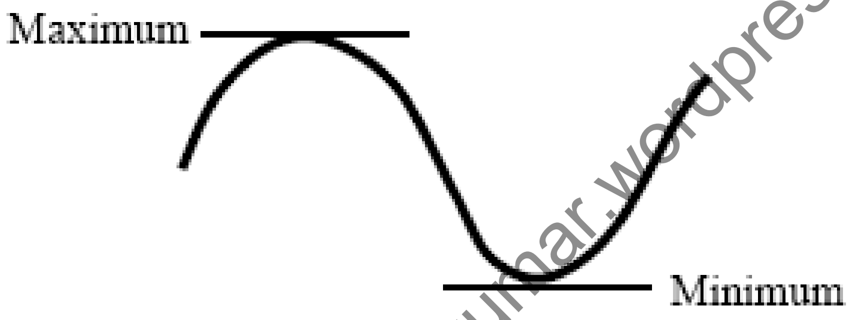
- Introduction to Optimization
- Particle Swarm Optimization (PSO) Algorithm
- Variants in PSO
- Ant colony Optimization
- Inspiration - Termites
- Development of Algorithm
- Parameter Study
- Tests

Optimization is Everywhere

- The more we know about something, the more we see where optimization can be applied.

- Some personal decision making
  - Finding fastest route home or class
  - Optimal allocation of time for home work
  - Optimal budgeting

Calculus



Maximum and minimum of a smooth function is reached at a stationary point where its gradient vanishes.

Goal of Optimization

Find values of the variables that minimize or maximize the objective function while satisfying the constraints.

Mathematical Formulation of Optimization Problems

minimize the objective function

$$\min f(x), x = \left(x_1, x_2, \ldots\ldots, x_n\right)$$

subject to constraints

$$c_i(x) \geq 0$$

$$c_i\left(x\right) = 0$$

Example

$$\min\left[\left(x_1 - 2\right)^2 + \left(x_2 - 1\right)^2\right]$$

$$\text{subject}: x_1{}^2 - x_2{}^2 \leq 0$$

$$x_1 + x_2 \leq 2$$

Classification of Optimization Methods

- Single variable
- Multi-variable
- Constrained
- Non-constrained
- Single objective
- Multi-objective
- Linear
- Non-linear

SWARMS

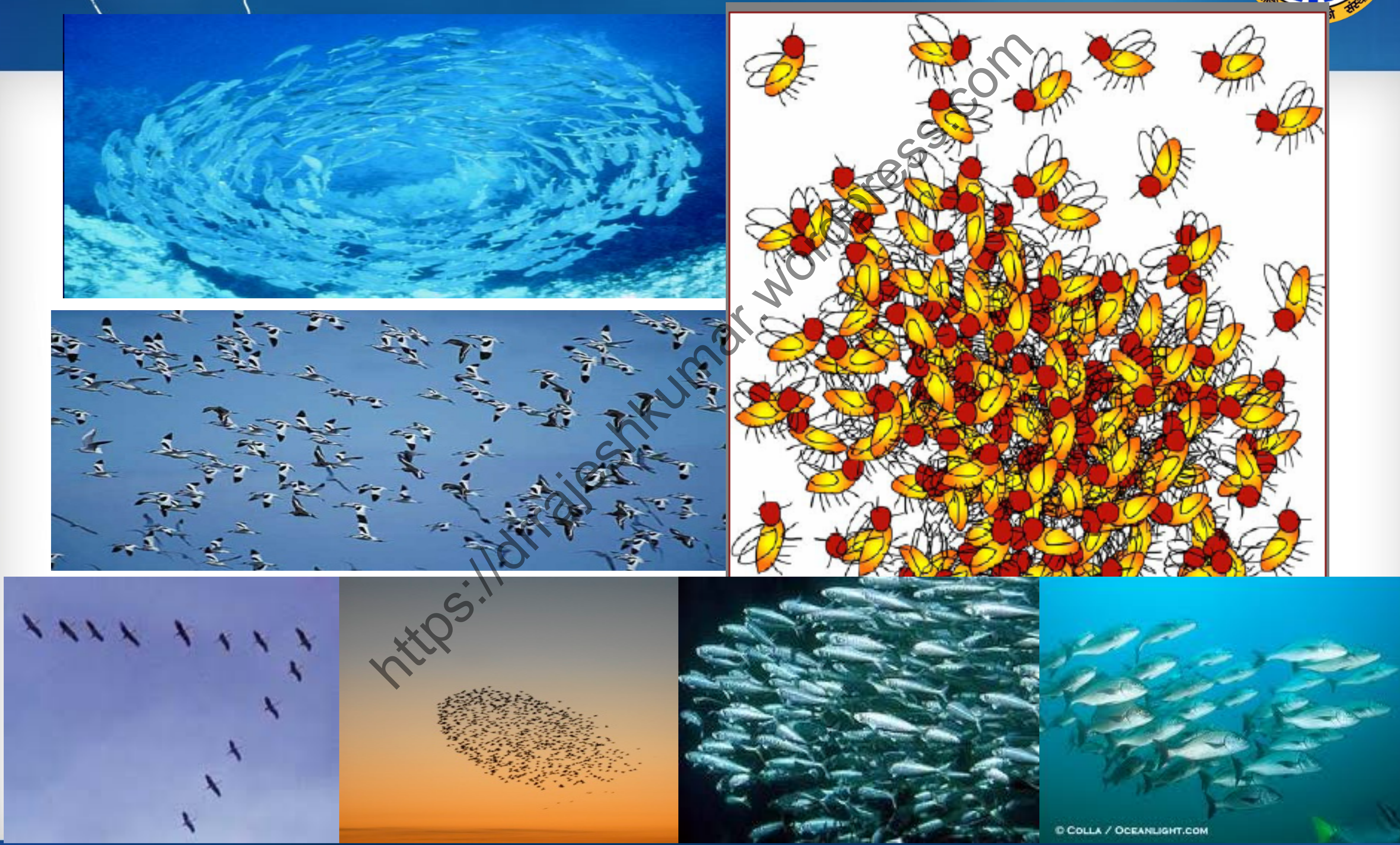


Figure 1: bird flocking and Fish schooling

The basic idea

- Each particle is searching for the optimum
- Each particle is *moving* (can't search otherwise!), and hence has a *velocity*.
- Each particle remembers the position it was in where it had its best result so far (its *personal best*)

- *But this would not be much good on its own; particles need help in figuring out where to search.*

The basic idea

- The particles in the swarm *co-operate*. They exchange information about what they' ve discovered in the places they have visited

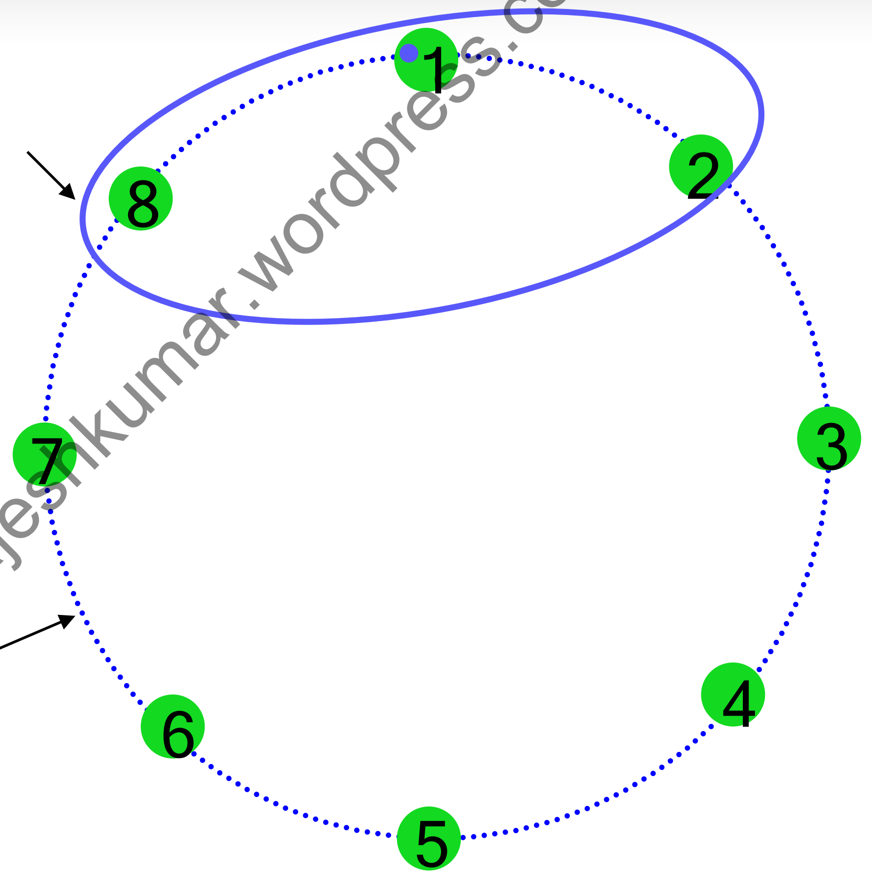
- The co-operation need only be very simple. In basic PSO (which is pretty good!) it is like this:

  - A particle has a *neighbourhood* associated with it.
  - A particle knows the fitnesses of those in its neighbourhood, and uses the *position* of the one with best fitness.
  - This position is simply used to adjust the particle's velocity

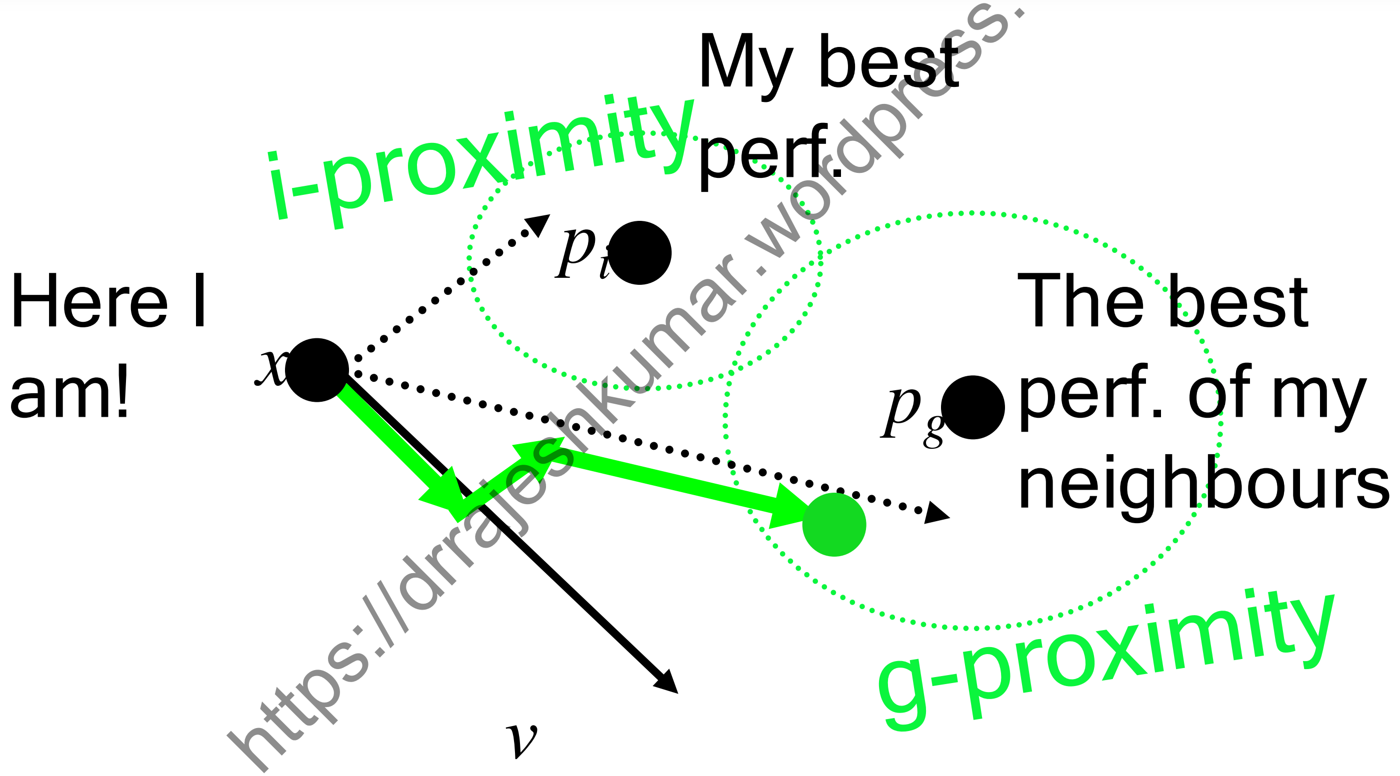
Neighborhoods

Particle 1's 3-neighbourhood

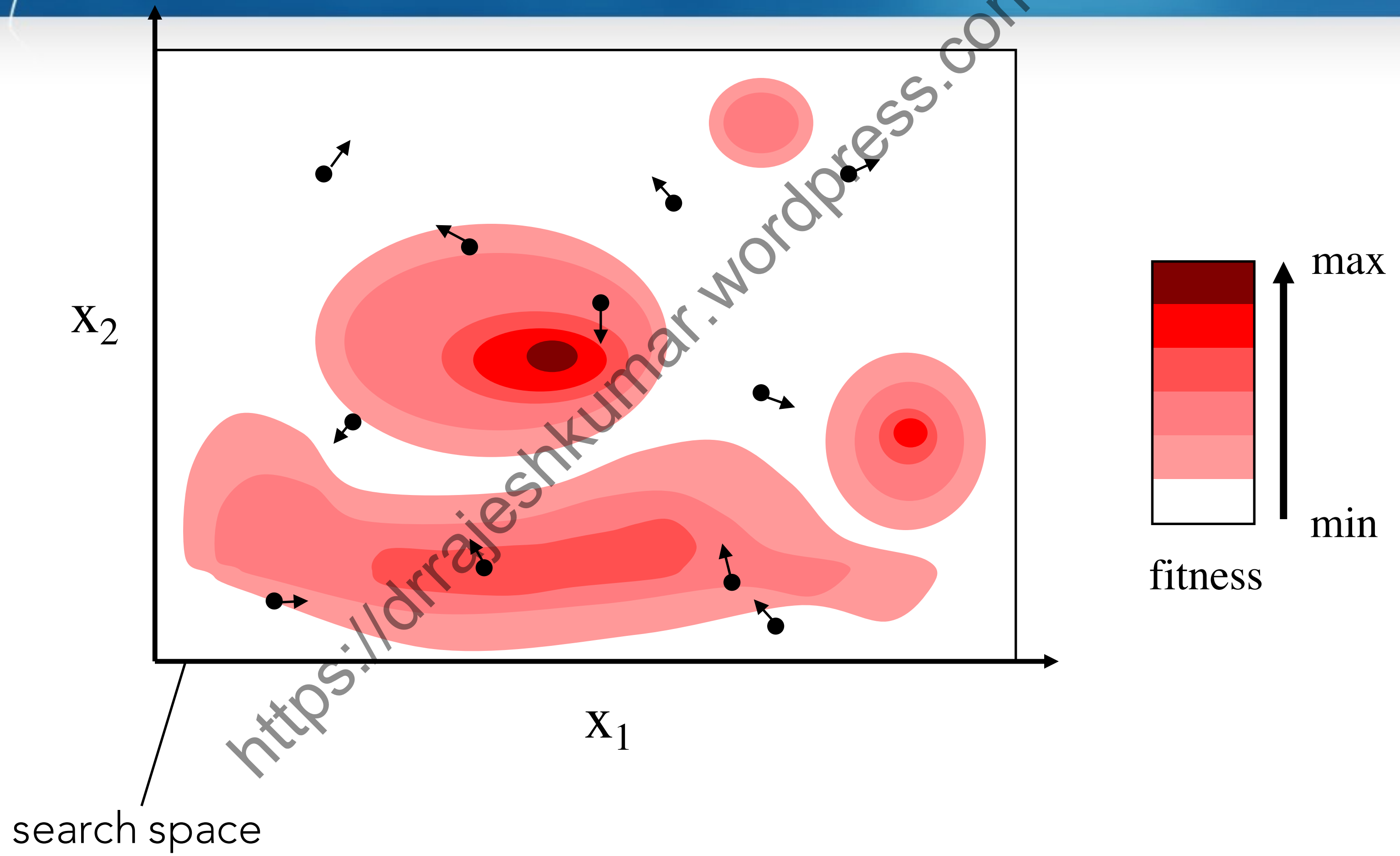
Virtual circle



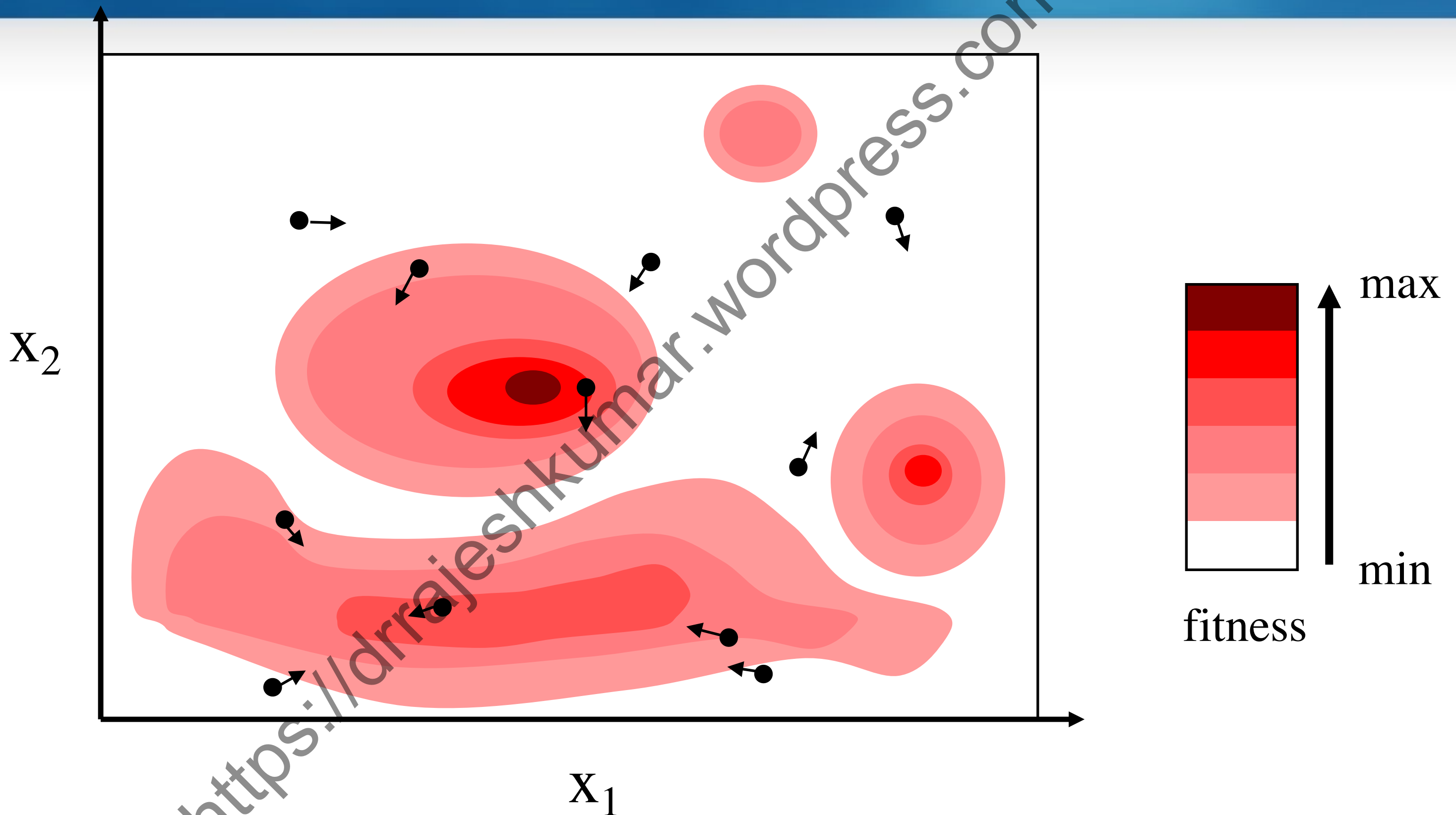
"Psychosocial compromise"



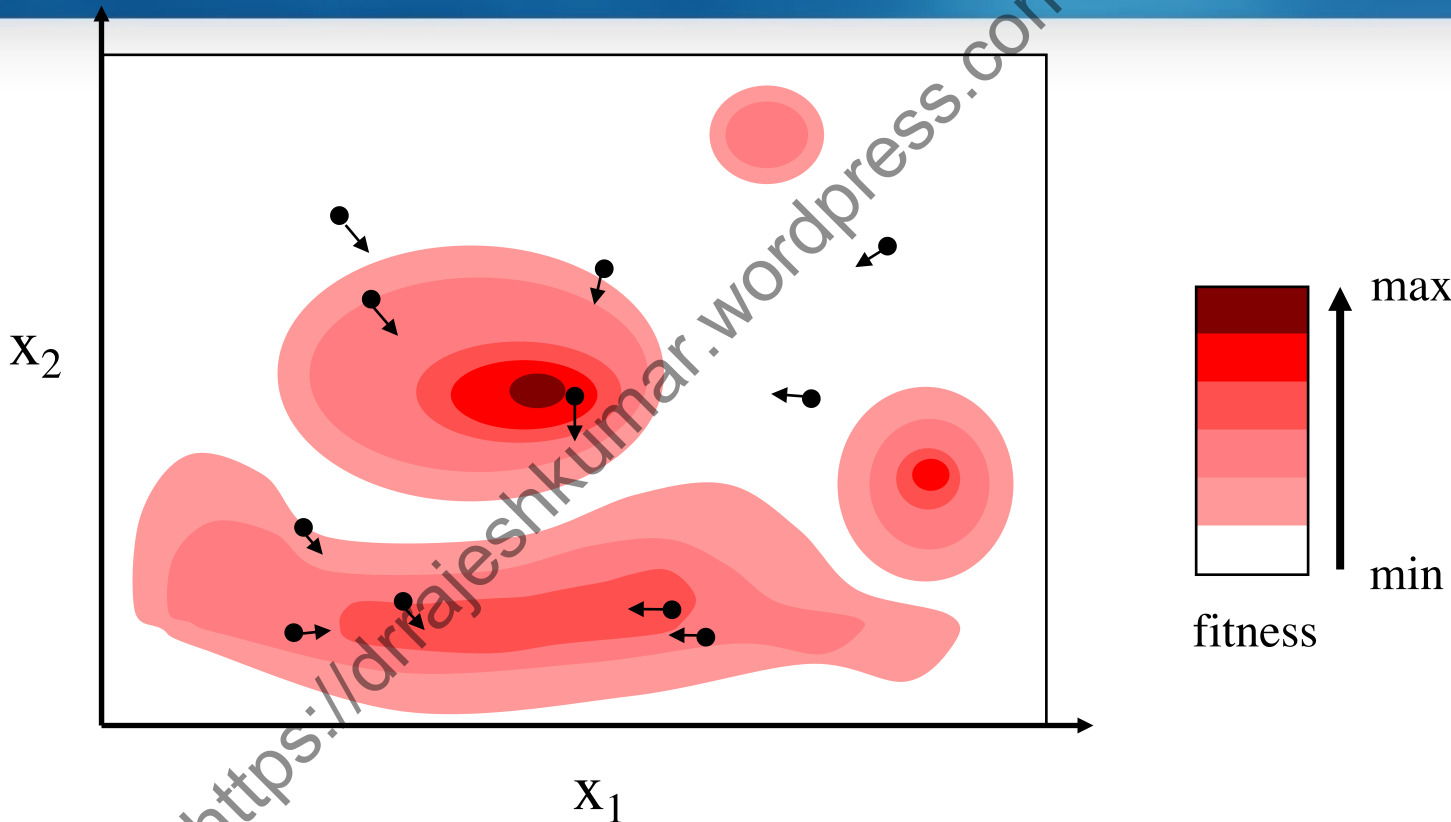
Particle Swarm Optimization – Animation



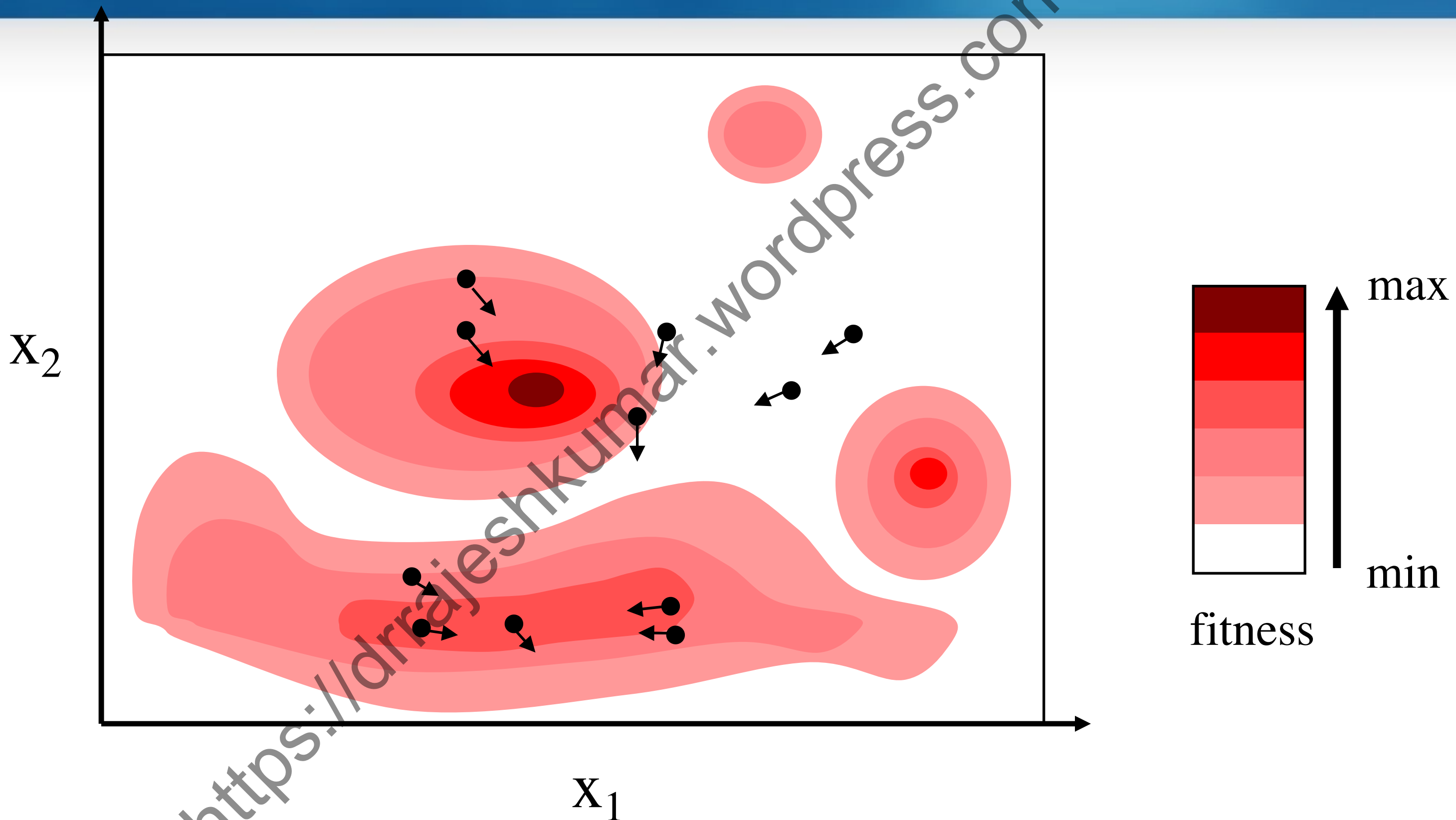
Particle Swarm Optimization – Animation



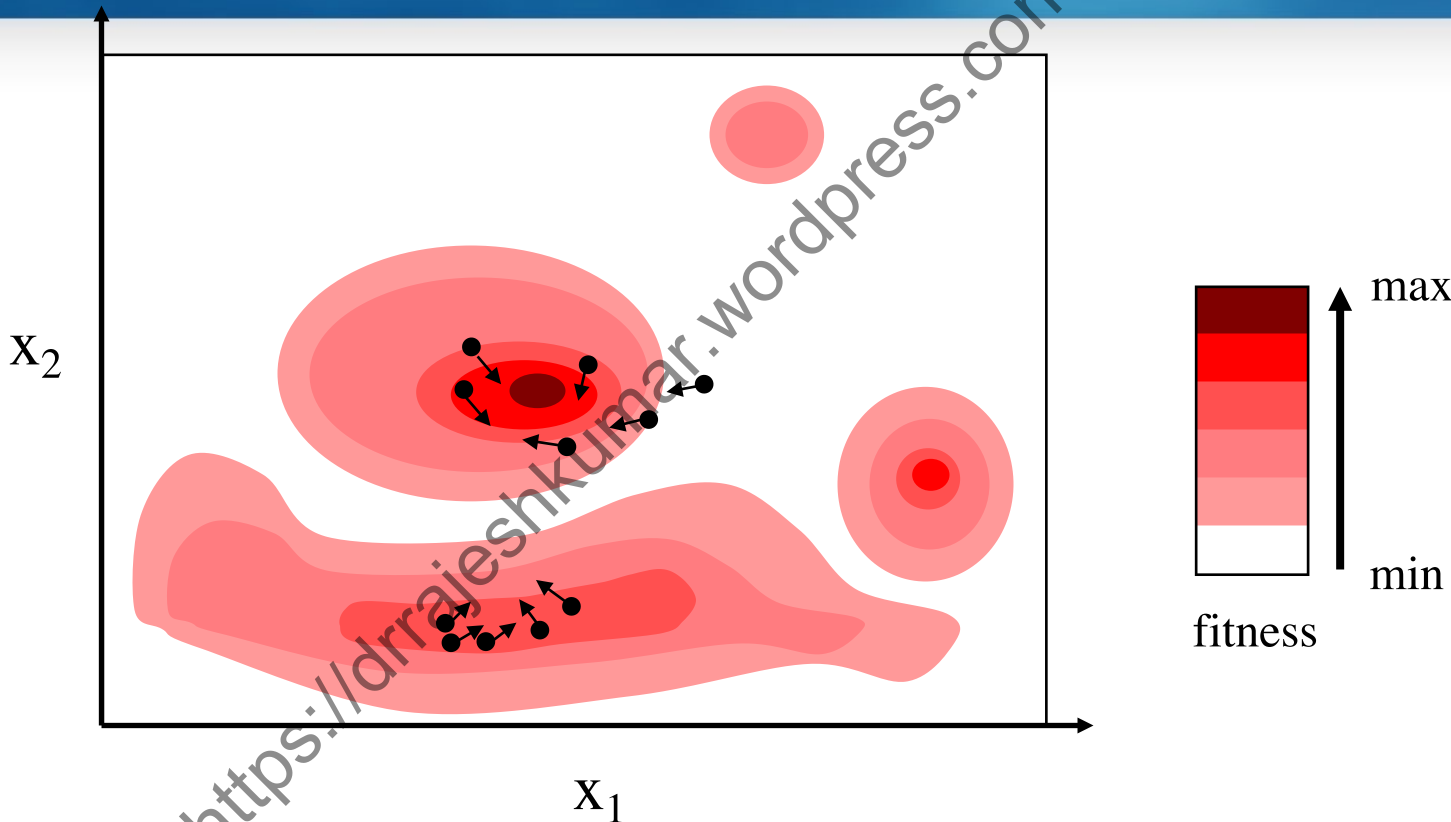
Particle Swarm Optimization – Animation



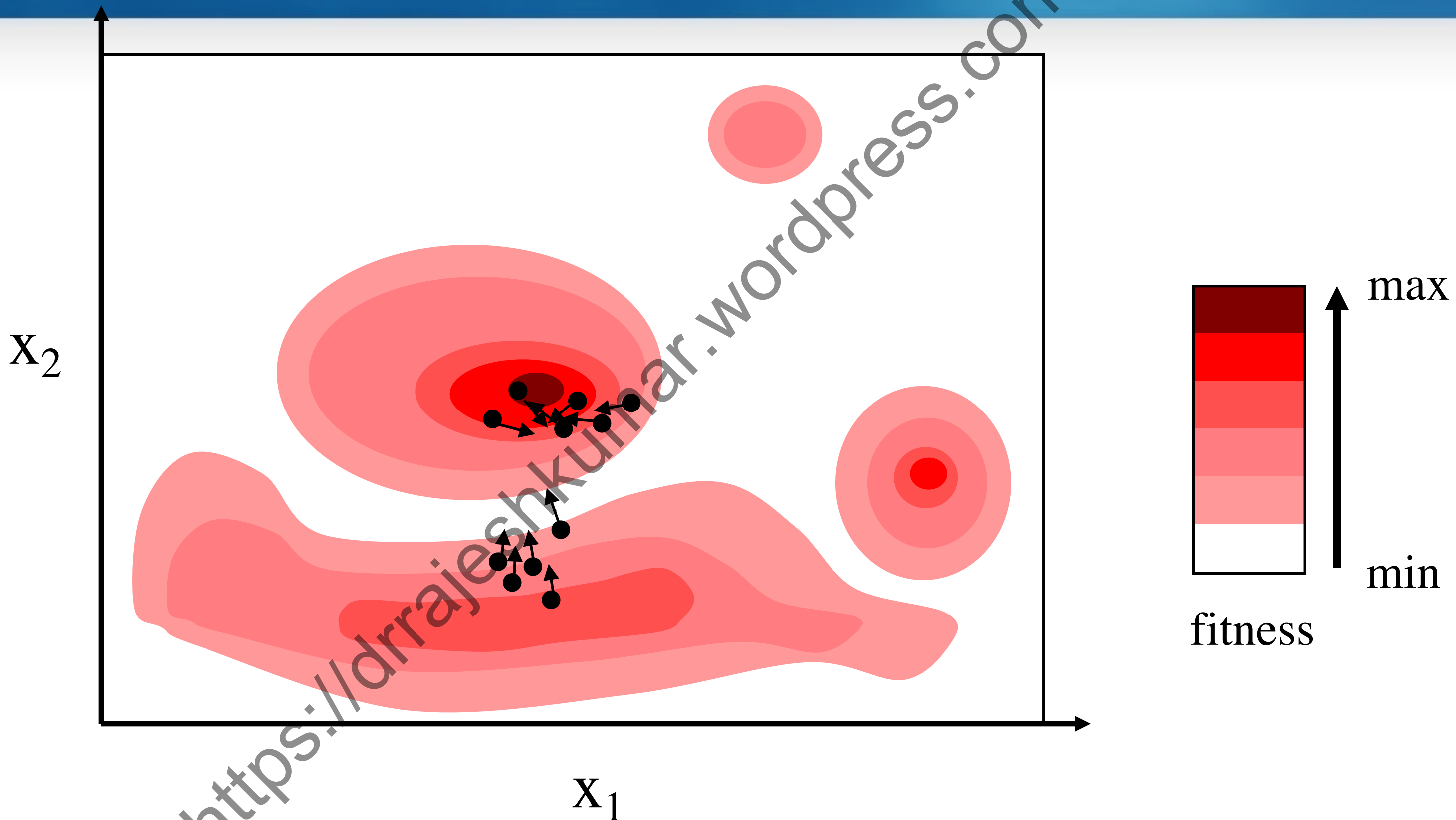
Particle Swarm Optimization – Animation



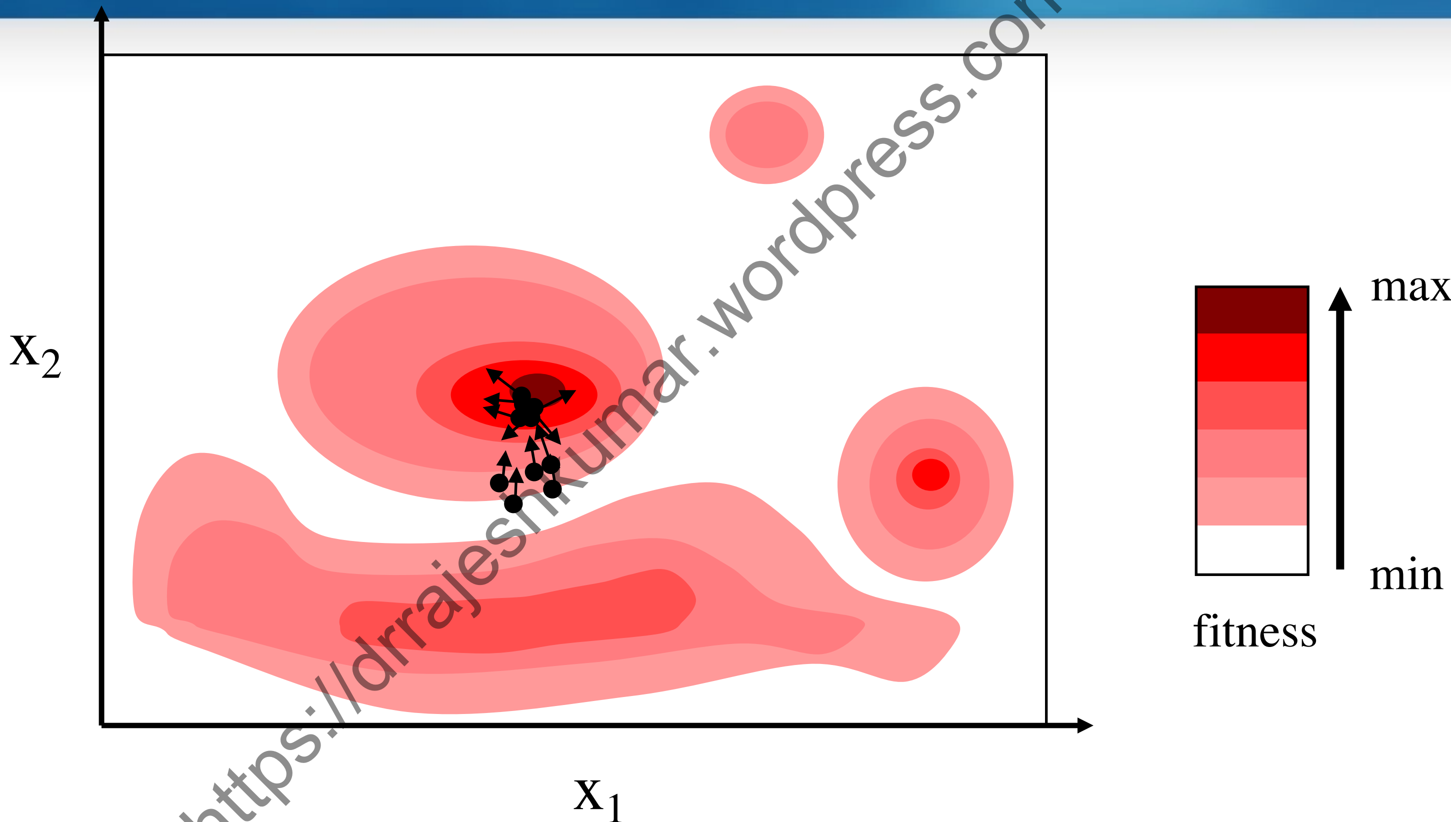
Particle Swarm Optimization – Animation



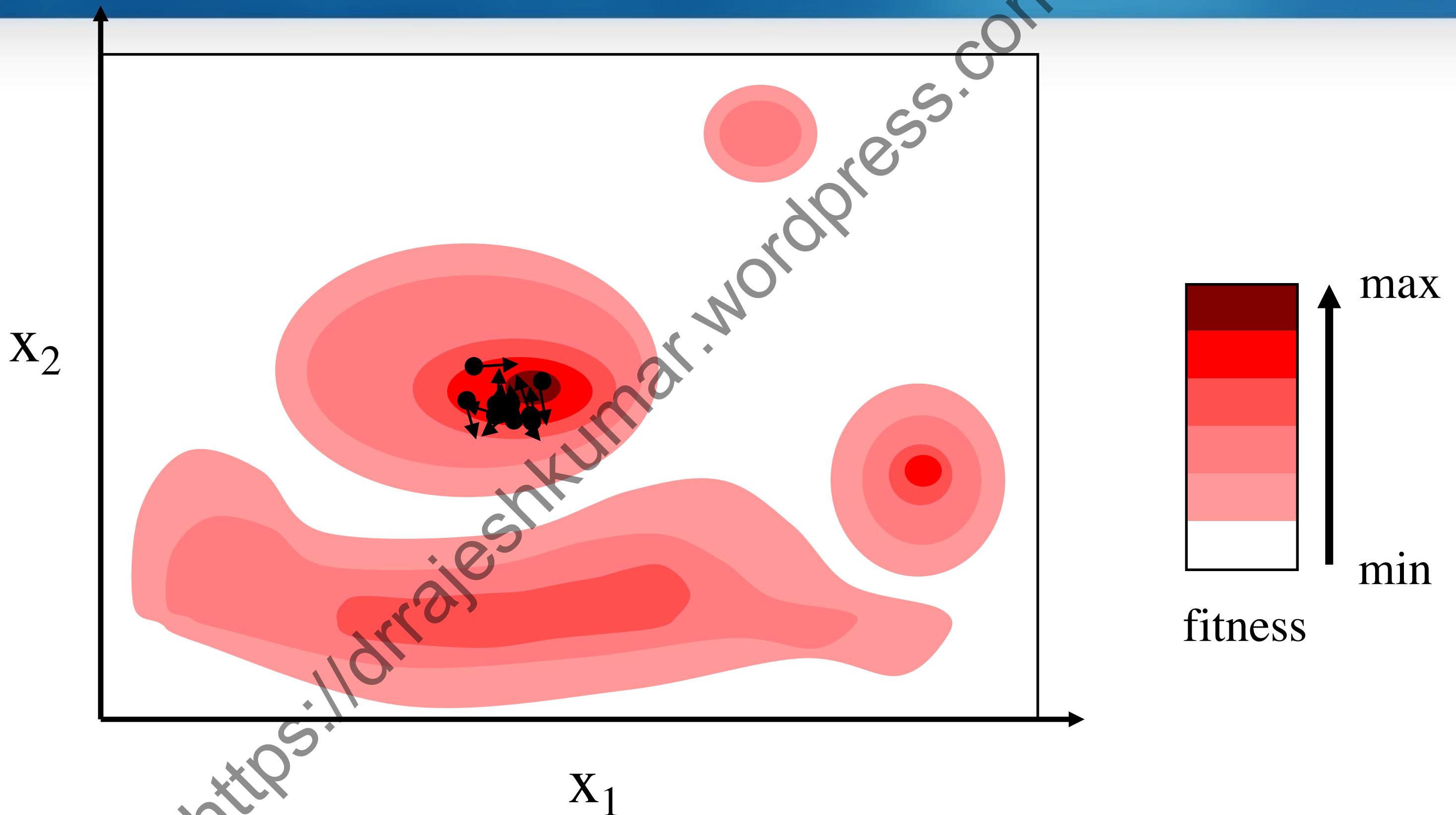
Particle Swarm Optimization – Animation



Particle Swarm Optimization – Animation



Particle Swarm Optimization – Animation



Particle Swarm Optimisation
Swarm search

Velocity calculation

$$v_{id(t)} = \omega v_{id(t-1)} + c_1 \times rand() \times (p_{id} - x_{id}) + c_2 \times Rand() \times (p_{gd} - x_{id})$$

Position update

$$x_{id(t)} = x_{id(t-1)} + v_{id(t)}$$

x_{id} – current value of the dimension “d” of the dividual “i”
v_{id} – current velocity of the dimension “d” of the individual “i”.
P_{id} – optimal value of the dimension “d” of the individual “i” so far.
P_{gd} – current optimal value of the dimension “d” of the swarm.
c_1, c_2 – acceleration coefficients.
ω - inertia weight factor

Related Issues

- There are a number of related issues concerning PSO:
  - Controlling velocities (determining the best value for Vmax),
  - Swarm Size,
  - Inertia weight factor,
  - Robust Settings for (C1 and C2),

The Algorithm

1. Initialise particles in the search space at random.
2. Assign random initial velocities for each particle.
3. Evaluate the fitness of each particle according a user defined objective function.
4. Calculate the new velocities for each particle.
5. Move the particles.
6. Repeat steps 3 to 5 until a predefined stopping criterion is satisfied.

Some Comments on C_1 C_2

- $c1 = c2 = 0$?
- $c1 > 0, c2 = 0$:

cognition-only model

- $c1 = 0, c2 > 0$:

social-only model

- $c1 = c2 > 0$:

particles are attracted towards the average of yi and ˆyi

- $c2 > c1$:

more beneficial for unimodal problems

- $c1 < c2$:

more beneficial for multimodal problems

- Low c1 and c2:

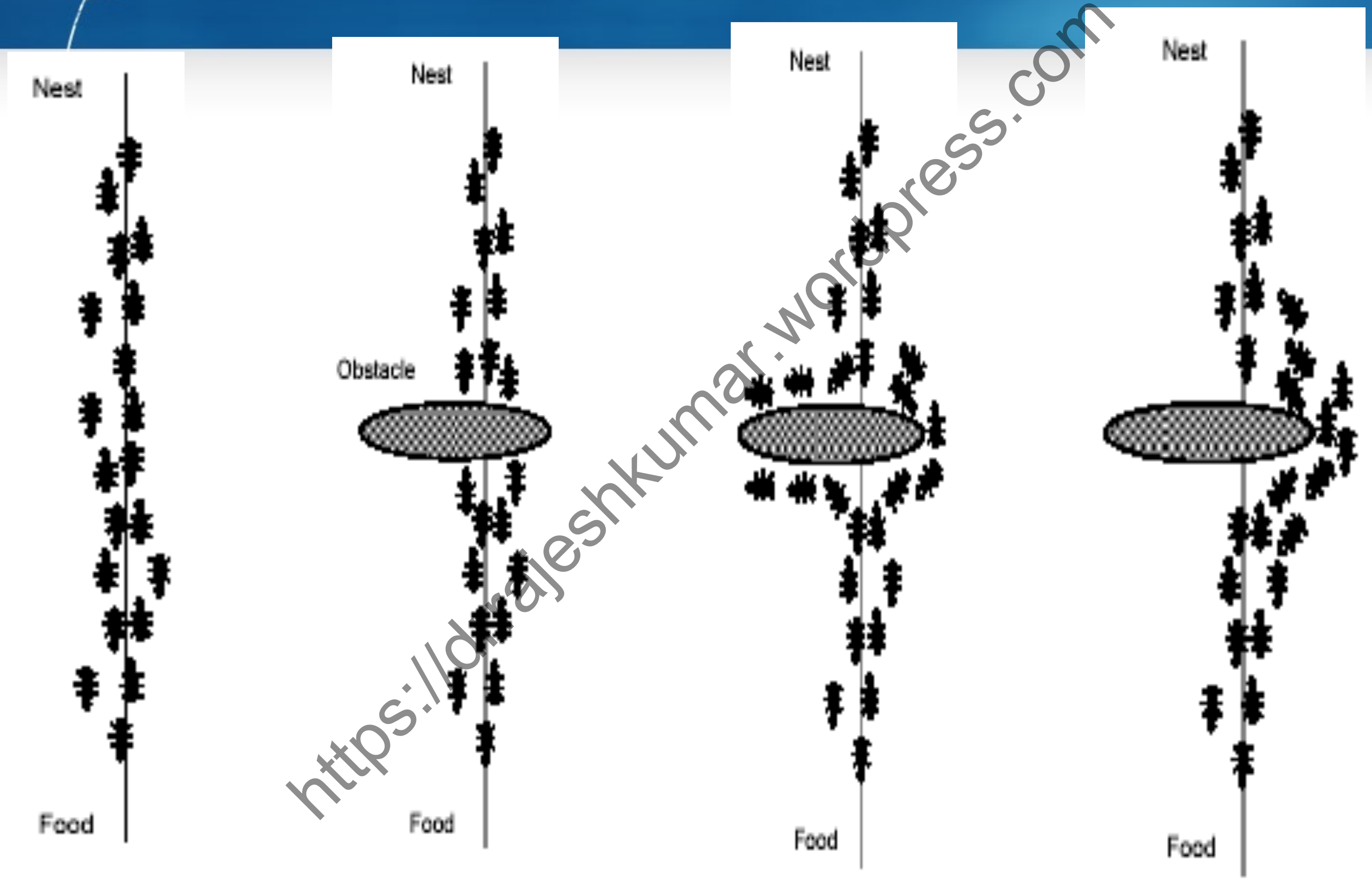
smooth particle trajectories

- High c1 and c2:

more acceleration, abrupt movements

- Adaptive acceleration coefficients
- $c1(t) = (c1,min - c1,max)\ t/nt + c1,max$
- $c2(t) = (c2,max - c2,min)\ t/nt + c2,min$

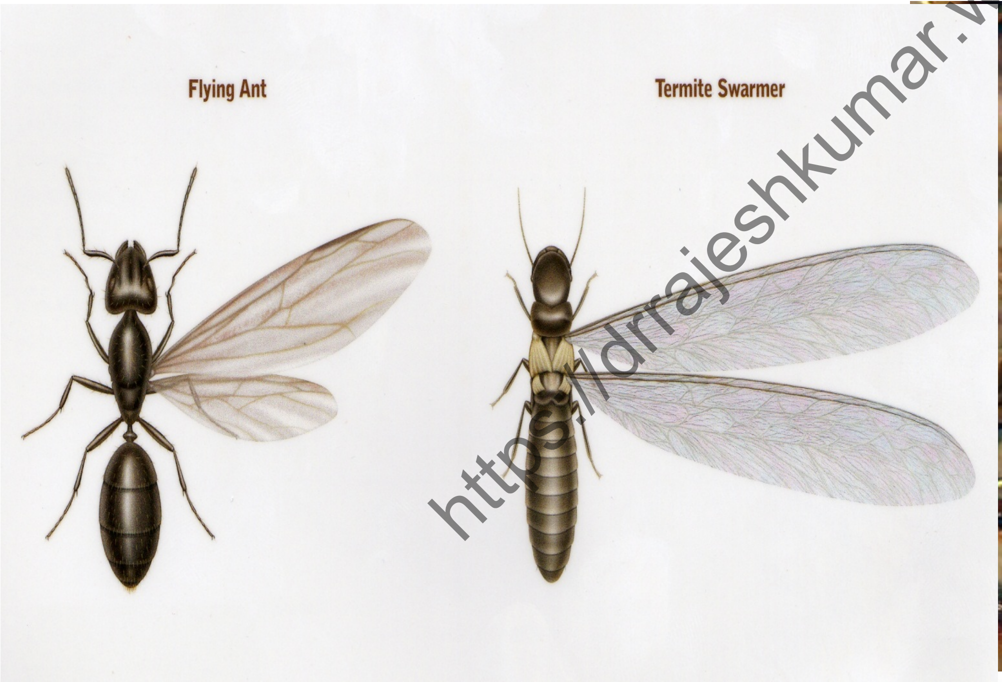
Natural behavior of ant



Ant Algorithms – (P.Koumoutsakos – based on notes L. Gamberdella (www.idsia.ch)

Termites

- 45 species of termites in US
  - Only 2 species are native



Termites

- Similar to ants, termites belong to a category of eusocial insects that work as decentralized entities in their groups.
- Termites are relatively simple insects of small size with a small number of neurons within them.
- This limits their ability to perform complex tasks at the individual level.
- On the other hand, a termite colony acts as an intelligent entity exhibiting a high level of self-organization and perform highly complex tasks.
- They exhibit relatively simple rules at the individual level to perform complex tasks at the group level without any form of centralized control.
- Their colonies can range from few hundred to few million in size.

- A typical termite colony consists of nymphs, workers, soldiers and reproductive individuals within it.
- To design any swarm intelligence inspired algorithm it is necessary to understand whether the given group replicate features of self-organization or not.
- In any decentralized systems, all the individual entities work on the basis of local information to achieve a common goal.
- Self-organization is a process in which with the help of some local interactions between the lower system of entities some coordination occurs in the working of the group.
- This process occurs with some sort of initial randomness and is not controlled by any external/central authority.
- This overall process takes place in a decentralized manner such that individual entities work on the basis of local information.

- Being a decentralized group, termite colony use a mechanism called stigmergy that is a kind of self-organization in which individuals in a group try to coordinate by indirect communication.
- Similar to ants, termite agents' use indirect communication by sensing and modifying the local environment.
- Termite workers build a nest by following a set of simple decentralized rules.
- Each worker takes a mud-ball from the environment and adds a small amount of pheromone to it.
- Then this mud-ball is initially placed probabilistically at a random position on the ground.
- But the probability of depositing a mud-ball at a particular place is increased with increase in the concentration of pheromone at that location.
- By mud-ball placing columns of nest gets formed. This process of nest building is similar to trail recruitment process as replicated within ant colonies.

- Various features exhibited by a self-organized system are
- **Positive and negative feedback**: These Information are taken from the output and applied back to the input of the system. Positive feedback tries to take system towards more stable state .
- **Fluctuations**: Fluctuations includes those process that introduces randomness into the system. For proper working of any self-organized system, these fluctuations are important as they help system to avoid stagnation and helps to maintain a proper balance between exploration and exploitation.
- **Multiple interactions**: This provides a mechanism for the system to have a combined intelligence by the process of learning through the information of individuals in the society.

Patterns




- *Cornitermes cumulans* termite workers are correlated to a certain extent. The individual steps in the termite walking were dependent on the previous steps with dependency getting decreased along the trajectory. For analyzing the correlation among the successive steps they used a correlation function (C.F.).

$$|C.F.(t)| = \frac{1}{(N-t_1)\sigma^2}\sum_{t=1}^{N-t_1}[l(t)-\bar{\theta}][l(t+t_1)-\bar{\theta}]$$

- The termite walking pattern was found to have long range correlations decaying as power laws.
- This step correlation exhibited by the termite workers became the inspiration for our work.
- The proposed algorithm tries to incorporate this step correlation by using greed based correlation multiplication factor selection.
- The algorithm does so by selecting a comparatively higher factor for the favorable regions.
- This process of selection of proper factor acts as a feedback to the system.
- Further evolutionary features were added to the algorithm to help the algorithm to avoid stagnation.

- Each individual in the swarm is called agent. Each agent has seven attributes/properties associated with it: -
- Position: It denotes the present position of the agent within the sample space
- Fitness: represent fitness or objective function value of the agent at
- Velocity: It denotes the amount of movement that the individual agent would undergo in the present iteration
- Best position & Fitness: It denotes the best position and fitness (in terms of fitness) that has been found by the agent so far.
- Best fitness: Represented by $pbestfit_i$, it denotes the fitness or objective function value of the agent ' *i*' at $pbest_i$.
- Past velocities: Past velocities of *the* agent.
- Correlation coefficient: It represents the weight of particular past velocity in the present velocity.

Phase -1:Attribute Update Phase

- This phase involves updating various attributes like fitness, past velocities, correlation coefficients, best position and best fitness of each agent
- At first fitness or objective function value of the agent at the current position is evaluated. This is followed by updating past velocity and correlation coefficient matrices
- If the present fitness value is better than the previous fitness of the agent than the CC(i,1) is updated to a higher value (CC_{high})
- This helps the agent to move more in the direction in which it finds better fitness values as compared to those directions in which it's fitness decreases.

Phase – 2: Replacement and Mutation Phase

- Agents with worst fitness are killed and replaced by new agents.
- R bad performing agents are killed and replaced with new agents by initializing them according to attributes of surviving agents in the swarm.
- This is followed by mutating one of the randomly selected dimensions of each new agent by using mutation equation

$$x_{i,j} = x_{i,j} + r_3 * [ub_j - lb_j]/2$$

- Although this phase can result in slightly reduced convergence rate it helps the algorithm to avoid stagnation condition.

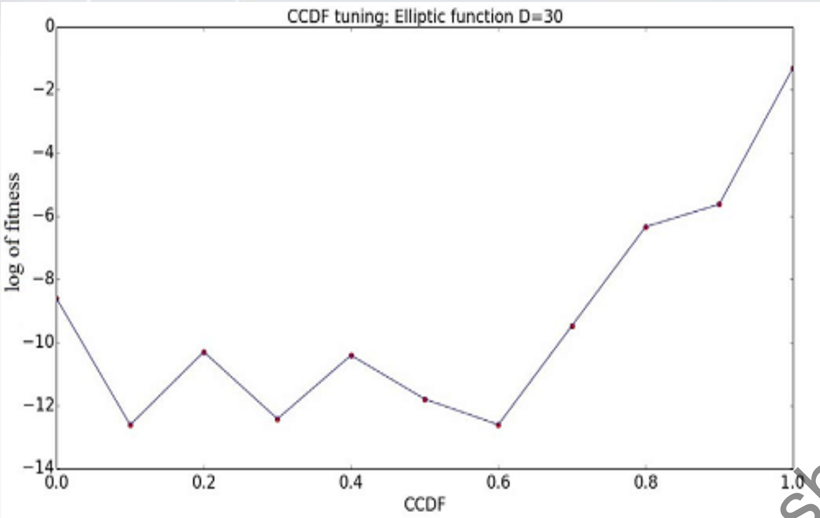
Phase – 3: Swarm Update Phase

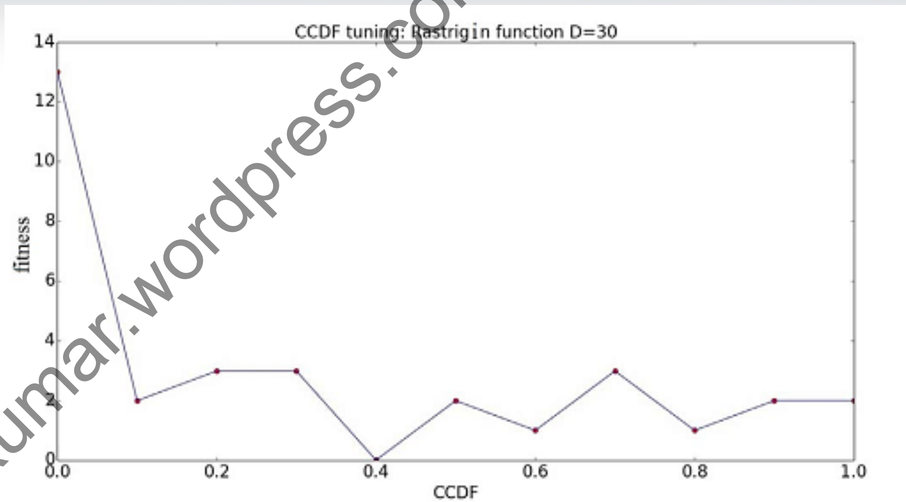
- This phase involves updating each agent's velocity and position in the search space so as to be used in the next iteration.
- Velocity update equation:-

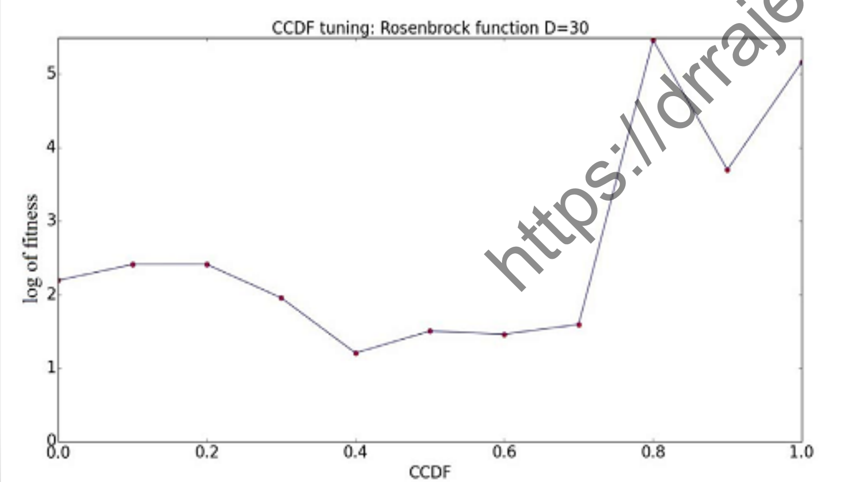
$$v_i = \sum_{t=1}^{t_{max}} [CC(i, t)*vpast_i^t] + c_1*r_1*(pbest_i - x_i) + c_2*r_2*(gbest - x_i)$$

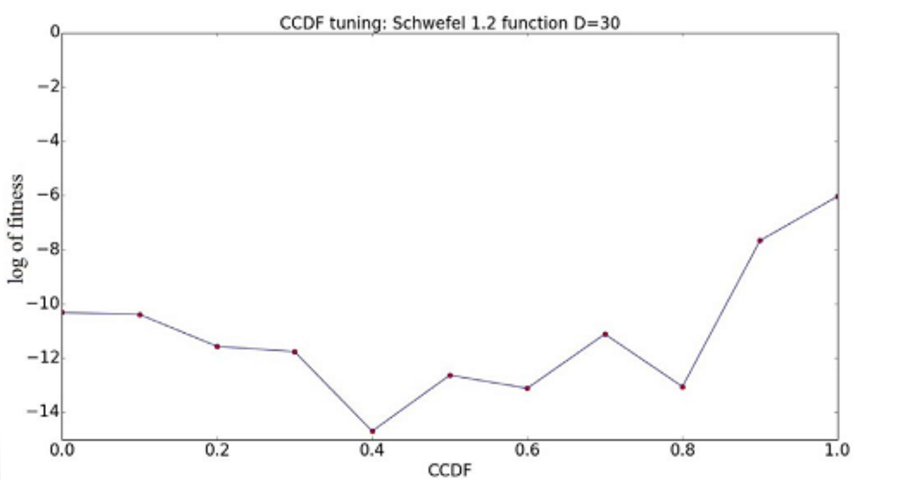
- The algorithm makes use of past tmax velocities in a greedy manner to formulate a correlation function as a replacement for weighted (inertia weights) velocity.

Impact of Hyper Parameters CCDF

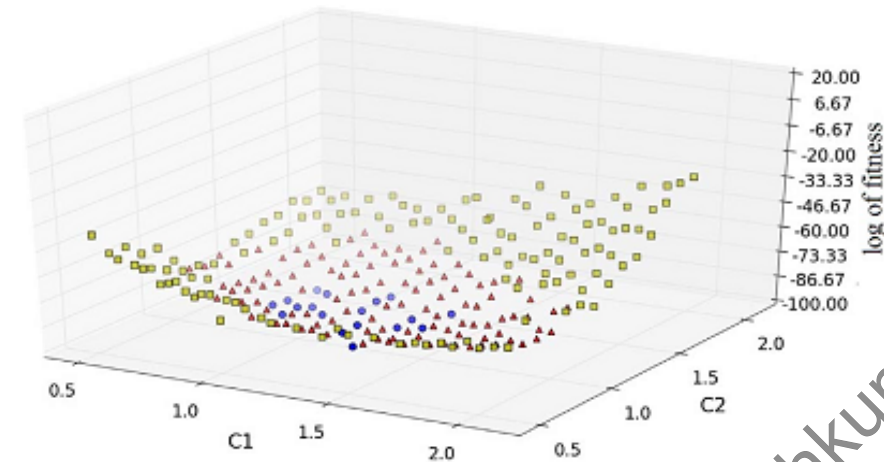




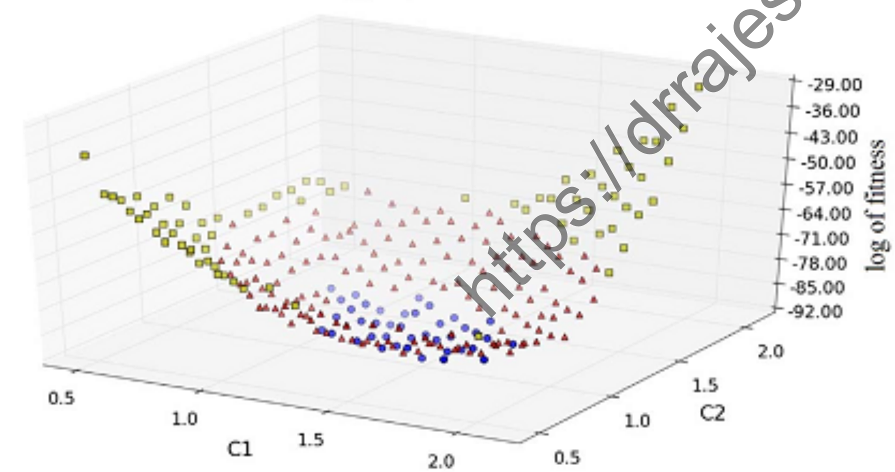




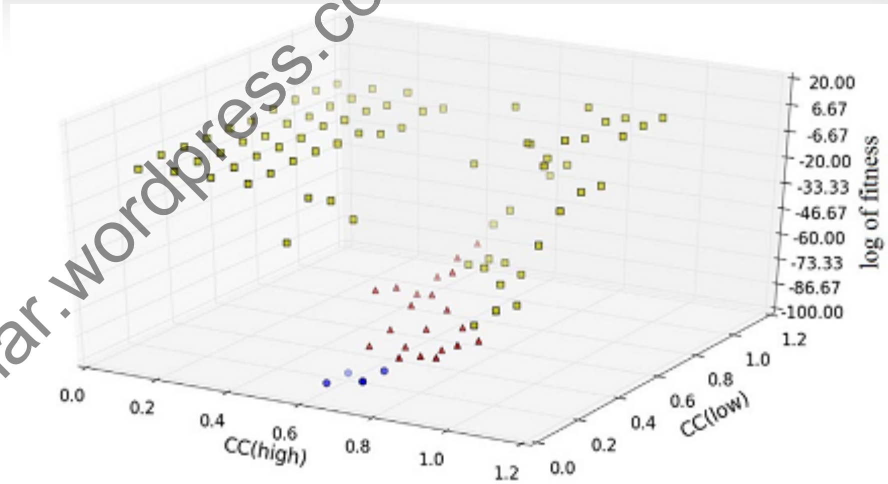
C1- C2 Tuning curves



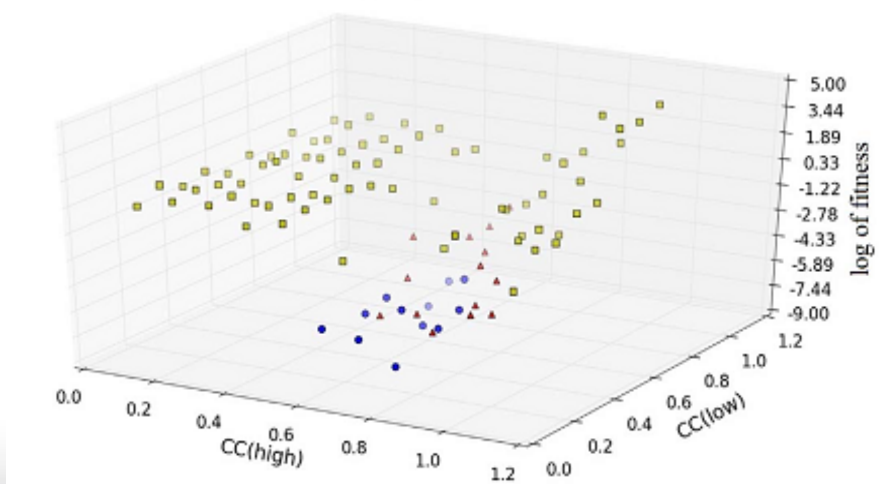
(a) Elliptic function D=10



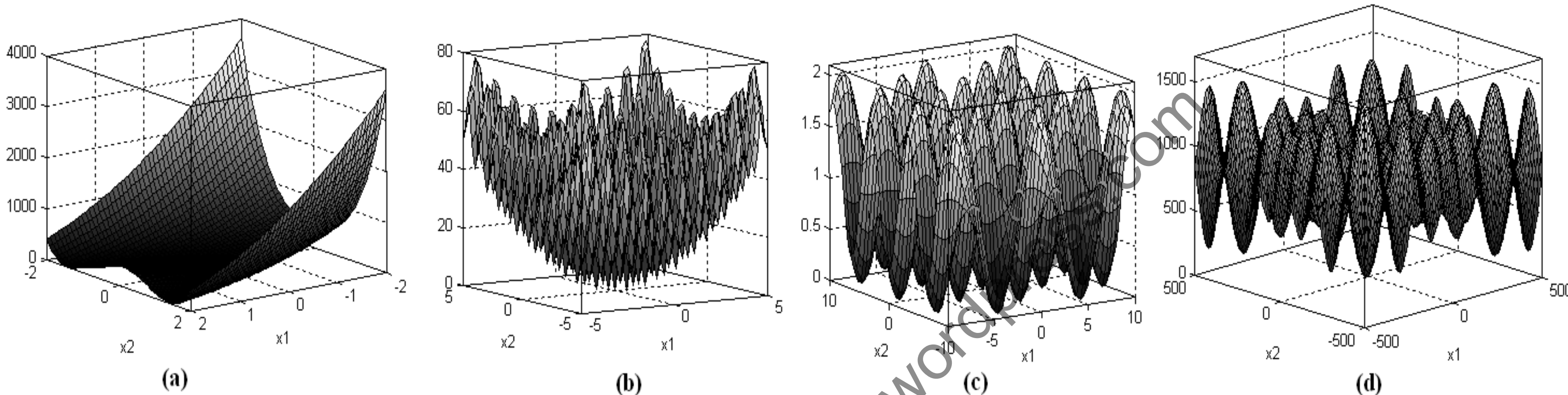
(b) Sphere function D=10



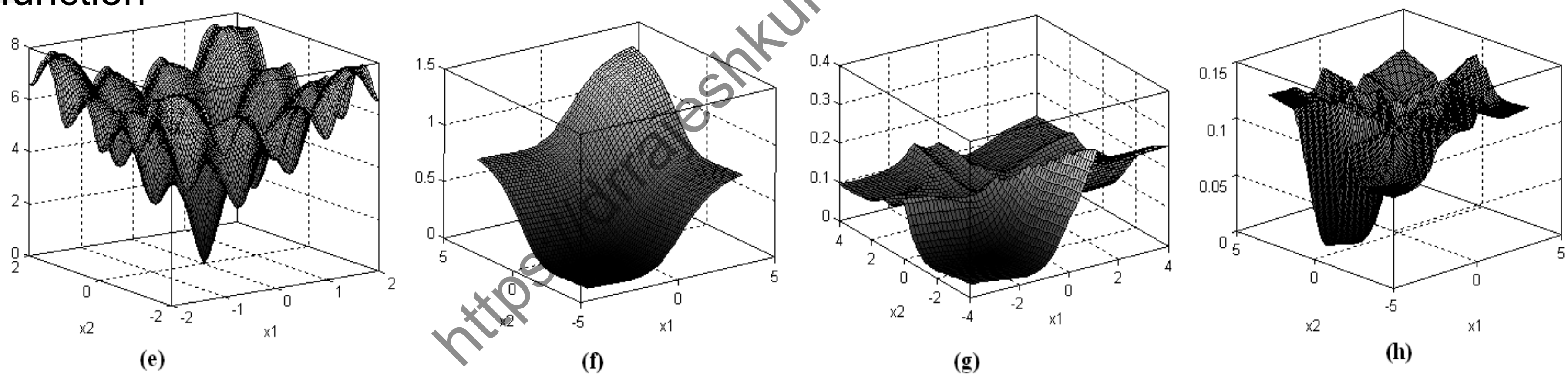
(a) Elliptic function D=10



Multi Modal Function



(a) Rosenbrock function, (b) Rastigrin function, (c) Grienwank function, (d) Schwefel function



(e) Ackley function, (f) Kennedy multimodal function generator (M=1 peaks),
(g) Kennedy multimodal function generator (M=10 peaks),
(h) Kennedy multimodal function generator (M=100 peaks).

Benchmark Functions

Function	D	Range	f_{min}	
Elliptic	30	[−100,100]	0	U
Sphere	30	[−100,100]	0	U
Greiwank	30	[−600,600]	0	M
Rastrigin	30	[−5.12,5.12]	0	M
Rosenbrock	30	[−5,10]	0	U
Schwefel 1.2	30	[−100,100]	0	U
Ackley	30	[−32,32]	0	M
Step	30	[−100,100]	0	U
Shifted Elliptic	30	[−100,100]	0	U
Shifted Ackley	30	[−32,32]	0	M
Beale	2	[−4.5,4.5]	0	U
Six Hump Camel Back	2	[−5,5]	−1.0316	M
Hartmann 3D	3	[0,1]	−3.8628	M
Hartmann 6D	6	[0,1]	−3.3224	M
Easom	2	[−100,100]	−1	U
Goldstein	2	[−2,2]	3	M
Schwefel 2.22	30	[−10,10]	0	U
Schwefel 2.21	30	[−100,100]	0	U
Schwefel	30	[−500,500]	0	M
Noise	30	[−1.28,1.28]	0	U
Penalised1	30	[−50,50]	0	M
Penalised2	30	[−50,50]	0	M
Shifted Rotated	30	[−100,100]	0	M
Rotated Greiwank	30	[−600,600]	0	M
Rotated	30	[−32,32]	0	M
Composition function 1	10	[−5,5]	0	M
Composition function 2	10	[−5,5]	0	M

Comparison with GWO , PSO , GSA,DE, ABC and FEP

F		TSC-PSO	GWO	PSO	GSA	DE	ABC	FEP
f1	Mean	**1.76E-108**	4.45E-24	2.33E-04	4.57E-15	7.87E-13	4.60E-09	2.48E-04
	StD	4.65E-109	2.73E-24	1.04E-04	2.78E-15	6.11E-13	7.80E-10	1.98E-04
	t(h)		-11.63(1)	-15.93(1)	-11.72(1)	-9.2(1)	-42.07(1)	-8.92(1)
f2	Mean	**5.07E-128**	7.46E-28	1.48E-04	2.94E-16	8.71E-14	7.32E-16	6.16E-04
	StD	1.71E-128	5.42E-28	4.38E-05	2.56E-16	2.64E-14	2.95E-16	3.85E-04
	t-test:t(h)		-9.82(1)	-24.17(1)	-8.2(1)	-21.89(1)	-17.7(1)	-11.43(1)
f3	Mean	1.27E-02	4.13E-03	7.88E-03	4.45E+01	**0.00E+00**	3.35E-13	1.59E-02
	StD	8.15E-03	3.13E-03	6.41E-03	3.77E+01	0.00E+00	1.74E-13	7.41E-03
	t value		6.97(1)	3.29(1)	-8.44(1)	11.09(1)	11.09(1)	-2.1(1)
f4	Mean	**1.28E-11**	2.83E-01	6.93E+01	3.06E+01	5.92E+01	1.35E-03	5.94E-02
	StD	2.14E-12	1.16E-01	2.54E+01	6.82E+00	3.71E+01	8.49E-04	1.18E-02
	t value		-17.44(1)	-19.49(1)	-32.05(1)	-11.41(1)	-11.37(1)	-35.92(1)
f5	Mean	**8.45E-14**	2.53E+01	1.50E+02	7.15E+01	0.00E+00	8.70E-02	4.54E+00
	StD	4.09E-14	5.71E+00	9.85E+01	6.12E+01	0.00E+00	6.63E-02	8.87E-01
	t value		-31.65(1)	-10.9(1)	-8.34(1)	14.75(1)	-9.38(1)	-36.58(1)
f6	Mean	**2.24E-122**	3.59E-06	7.84E+01	9.98E+02	7.15E-11	3.84E-12	1.70E-02
	StD	6.83E-123	9.23E-07	3.06E+01	6.50E+02	2.85E-11	2.12E-12	5.22E-03
	t value		-27.77(1)	-18.27(1)	-10.96(1)	-17.92(1)	-12.93(1)	-23.24(1)
f7	Mean	**9.87E-13**	1.43E-13	4.65E-01	5.26E-02	1.16E-07	1.73E-09	2.63E-02
	StD	2.62E-13	8.36E-14	2.06E-01	2.67E-02	8.14E-08	7.76E-10	1.73E-02
	t value		22.7(1)	-16.12(1)	-14.1(1)	-10.14(1)	-15.91(1)	-10.83(1)
f8	Mean	0.00E+00	1.25E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	StD	0.00E+00	6.44E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	t value		-13.88(1)	0(0)	0(0)	0(0)	0(0)	0(0)
f9	Mean	**0.00E+00**	4.05E-27	8.22E-06	7.67E-21	7.51E-18	5.03E-15	6.27E-14
	StD	0.00E+00	1.73E-27	6.10E-06	6.75E-21	2.77E-18	2.49E-15	3.08E-14
	t value		-16.7(1)	-9.62(1)	-8.12(1)	-19.34(1)	-14.43(1)	-14.54(1)
f10	Mean	**1.99E-14**	2.98E+01	1.56E+01	1.55E+01	2.87E-05	1.12E-04	2.67E+01
	StD	1.37E-14	2.24E+01	1.32E+01	8.27E+00	1.58E-05	6.24E-05	1.97E+01
	t value		-9.52(1)	-8.4(1)	-13.37(1)	-12.97(1)	-12.8(1)	-9.66(1)
f11	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	StD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	t value		0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
f12	Mean	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	StD	0.00E+00	0.00E+00	5.71E-15	4.48E-16	2.71E-13	3.46E-04	8.91E-08
	t value		0(0)	0(0)	0(0)	0(0)	-21.78(1)	0(0)

Comparison with GWO , PSO , GSA,DE, ABC and FEP

F		TSC-PSO	GWO	PSO	GSA	DE	ABC	FEP
f13	Mean	−3.86E+00	−3.86E+00	−3.86E+00	−3.86E+00	−3.86E+00	−3.85E+00	−3.86E+00
	StD	0.00E+00	4.14E−05	3.58E−14	7.11E−14	1.71E−14	0.00714	8.40E−06
	t value		0(0)	0(0)	0(0)	0(0)	−8.57(1)	0(0)
f14	Mean	−3.29E+00	−3.29E+00	−3.27E+00	−3.32E+00	−3.29E+00	−3.32E+00	−3.27E+00
	StD	0.057582013	0.001221	0.060516	0.023081	0.045455	2.25E−06	0.059
	t value		0(0)	−1.04(0)	2.17(1)	0(0)	2.77(1)	−0.91(0)
f15	Mean	−1.00E+00	−1.00E+00	−1.00E+00	−1.00E+00	−1.00E+00	−1.00E+00	−1.00E+00
	StD	0.00E+00	0.00E+00	2.17E−14	0.00E+00	6.16E−15	3.64E−07	0.00E+00
	t value		0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
f16	Mean	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.09E+00	3.02E+00
	StD	0.00E+00	3.12E−06	1.33E−15	4.17E−15	2.00E−15	1.14E−01	9.87E−02
	t value		−17.68(1)	0(0)	0(0)	0(0)	−9.7(1)	−75.12(1)
f17	Mean	**1.37E−37**	8.21E−17	4.13E−02	5.30E−02	1.65E−09	8.09E−11	9.70E−03
	StD	9.18E−38	3.53E−17	2.22E−02	1.47E−02	1.22E−09	1.92E−11	6.44E−03
	t value		−16.62	−13.31(1)	−26.73(1)	−9.71(1)	−30.13(1)	−10.75(1)
f18	Mean	**4.70E−11**	6.50E−07	1.25E+00	8.37E+00	0.00E+00	1.14E+01	3.03E−01
	StD	3.21E−11	3.59E−07	2.51E−01	4.05E+00	0.00E+00	9.63E+00	2.17E−01
	t value		−12.95(1)	−35.62(1)	−14.78(1)	10.46(1)	−8.47(1)	−10.01(1)
f19	Mean	**0.00E+00**	9.52E+03	1.24E+04	8.85E+03	2.64E+03	1.09E+02	1.28E+01
	StD	0.00E+00	7.24E+03	6.41E+03	5.14E+03	1.70E+03	2.06E+01	7.96E+00
	t value		−9.39(1)	−13.81(1)	−12.29(1)	−11.06(1)	−37.88(1)	−11.44(1)
f20	Mean	**1.94E−03**	2.56E−03	1.35E−01	1.08E−01	4.69E−03	1.38E−01	1.44E−01
	StD	1.00E−03	5.43E−04	1.13E−01	2.34E−02	3.61E−03	9.27E−02	3.56E−02
	t value		−3.95(1)	−8.35(1)	−32.35(1)	−5.25(1)	−10.49(1)	−28.52(1)
f21	Mean	**1.79E−32**	7.68E−02	1.23E−02	2.74E+00	8.75E−15	4.37E−14	1.07E−05
	StD	8.69E−33	4.24E−02	6.55E−03	6.92E−01	1.77E−15	1.38E−14	6.18E−06
	t value		−12.95(1)	−13.42(1)	−28.24(1)	−35.33(1)	−22.59(1)	−12.38(1)
f22	Mean	**1.62E−30**	6.51E−01	5.38E−03	1.03E+01	5.38E−14	6.63E−15	1.79E−04
	StD	1.18E−30	5.26E−01	4.31E−03	4.83E+00	1.92E−14	4.91E−15	1.28E−04
	t value		−8.85(1)	−8.93(1)	−15.23(1)	−20.04(1)	−9.64(1)	−10(1)
f26	Mean	**1.25E+02**	1.36E+02	3.23E+02	1.93E+02	3.24E+02	3.30E+02	2.72E+02
	StD	8.25E+01	8.91E+01	2.69E+02	4.43E+01	2.81E+02	2.08E+02	1.25E+02
	t value		−0.64(0)	−5(1)	−5.12(1)	−4.86(1)	−6.53(1)	−6.96(1)
f27	Mean	1.08E+02	1.14E+02	8.37E+01	2.00E+02	**1.19E+01**	1.52E+02	1.80E+02
	StD	2.18E+01	4.52E+01	5.04E+01	1.24E+02	3.45E+00	1.10E+02	3.64E+01
	t value		−0.9(0)	3.16(1)	−5.19(1)	31.07(1)	−2.82(1)	−12.12(1)

Shapiro-Wilk test for TSC-PSO, GWO [5], PSO [23], GSA [15], DE [13], ABC [26] and FEP [14] samples.

F	TSC-PSO	GWO	PSO	GSA	DE	ABC	FEP
f1	1.47E-01(0)	1.44E-01(0)	5.21E-03(1)	4.25E-03(1)	1.21E-02(1)	6.08E-03(0)	1.30E-01(0)
f2	5.14E-02(0)	1.02E-01(0)	2.20E-01(0)	9.89E-03(1)	3.61E-02(1)	2.98E-02(1)	1.99E-03(1)
f3	6.34E-02(0)	1.15E-01(0)	2.34E-02(1)	2.12E-01(0)	7.55E-02(0)	3.19E-01(0)	1.26E-02(1)
f4	9.35E-02(0)	3.29E-02(1)	2.92E-04(1)	5.68E-02(0)	6.21E-03(1)	8.81E-03(1)	5.27E-02(0)
f5	4.96E-02(0)	7.56E-02(0)	4.53E-02(1)	7.74E-02(0)	5.78E-02(0)	1.36E-01(0)	9.98E-03(1)
f6	7.81E-02(0)	8.75E-03(1)	1.95E-02(1)	1.11E-01(0)	2.69E-02(1)	9.60E-03(1)	9.52E-02(0)
f7	5.66E-03(1)	3.85E-03(1)	5.44E-03(1)	5.72E-04(1)	1.11E-01(0)	1.12E-03(1)	1.58E-02(1)
f8	8.11E-02(0)	5.98E-02(0)	5.23E-02(0)	2.41E-01(0)	1.36E-01(0)	7.87E-02(0)	1.54E-01(0)
f9	4.14E-02(1)	8.77E-02(0)	4.48E-02(1)	6.45E-02(0)	2.74E-03(1)	4.72E-02(1)	2.29E-03(1)
f10	2.98E-03(1)	1.49E-02(1)	5.67E-03(1)	1.89E-01(0)	2.90E-01(0)	4.29E-02(1)	5.07E-04(1)
f11	1.74E-01(0)	7.41E-02(0)	5.44E-02(0)	7.44E-02(0)	7.44E-02(0)	1.44E-01(0)	1.55E-[illegible](0)
f12	3.47E-01(0)	1.74E-01(0)	6.11E-02(0)	5.11E-02(0)	1.41E-01(0)	1.69E-02(1)	2.4[illegible]E-[illegible]6(1)
f13	1.11E-01(0)	4.08E-04(1)	7.44E-02(0)	6.77E-02(0)	6.45E-02(0)	4.7[illegible]E-02(1)	1.92E-03(1)
f14	7.44E-02(0)	1.18E-02(1)	7.06E-02(0)	2.30E-01(0)	5.93E-02(0)	[illegible].52E-10(1)	5.83E-04(1)
f15	2.14E-01(0)	4.14E-01(0)	7.89E-02(0)	8.77E-02(0)	1.2[illegible]E-[illegible](0)	7.58E-14(1)	1.24E-01(0)
f16	4.14E-01(0)	4.55E-05(1)	1.55E-01(0)	5.44E-02([illegible])	6.70E-02(0)	9.09E-03(1)	9.08E-02(0)
f17	7.41E-02(0)	2.19E-03(1)	3.06E-01(0)	1.06E-01(0)	2.56E-02(1)	2.21E-02(1)	6.90E-03(1)
f18	8.03E-02(0)	4.57E-03(1)	1.28E-01(0)	2.68E-03(1)	7.44E-02(0)	6.55E-03(1)	2.35E-03(1)
f19	7.71E-02(0)	1.15E-02(1)	[illegible]E-[illegible](1)	8.70E-02(0)	1.07E-03(1)	1.56E-01(0)	2.48E-04(1)
f20	3.73E-03(1)	4.17E-02(1)	5.00E-03(1)	8.38E-03(1)	4.73E-02(1)	2.89E-02(1)	2.01E-02(1)
f21	1.14E-01(0)	5.49E-03(1)	6.82E-03(1)	3.99E-02(1)	7.12E-03(1)	2.46E-01(0)	3.43E-02(1)
f22	8.14E-02(0)	1.24E-03(1)	9.95E-03(1)	1.99E-01(0)	5.86E-02(0)	5.47E-02(0)	8.13E-04(1)
f26	8.46E-02(0)	2.57E-02(1)	2.97E-02(1)	3.39E-02(1)	3.01E-01(0)	1.90E-02(0)	2.28E-02(1)
f27	1.67E-02(1)	7.88E-02(0)	2.62E-02(1)	1.80E-02(1)	2.34E-04(1)	1.52E-02(1)	4.05E-02(1)

Wilcoxon test against GWO [5], PSO [23], GSA [15], DE [13], ABC [26] and FEP [14].

F	GWO	PSO	GSA	DE	ABC	FEP
f1	1.05E-15 (1)	3.21E-18 (1)	1.05E-15 (1)	2.61E-12 (1)	3.21E-18 (1)	2.61E-12 (1)
f2	2.61E-12 (1)	3.21E-18 (1)	2.33E-09 (1)	3.21E-18 (1)	2.16E-13 (1)	1.48E-07 (1)
f3	1.12E-08 (1)	4.89E-03 (1)	2.33E-09 (1)	1.05E-15 (1)	1.05E-15 (1)	5.11E-02 (0)
f4	3.21E-18 (1)	1.56E-06 (1)	1.66E-17 (1)	6.14E-17 (1)	3.21E-18 (1)	3.21E-18 (1)
f5	4.22E-07 (1)	1.05E-1[illegible] (1)	2.61E-12 (1)	3.01E-14 (1)	2.61E-12 (1)	1.23E-12 (1)
f6	1.58E-12 (1)	1.4[illegible]E-15 (1)	1.59E-14 (1)	7.58E-09 (1)	3.21E-18 (1)	3.21E-18 (1)
f7	1.48E-14 (1)	3.21E-18 (1)	3.11E-17 (1)	2.82E-11 (1)	2.61E-12 (1)	6.14E-17 (1)
f8	4.[illegible]E-08 (1)	6.85E-01 (0)	5.15E-01 (0)	4.58E-01 (0)	4.55E-01 (0)	5.16E-01 (0)
f9	[illegible]E-15 (1)	2.71E-10 (1)	2.33E-09 (1)	7.58E-09 (1)	6.14E-17 (1)	3.21E-18 (1)
f10	2.16E-13 (1)	2.61E-12 (1)	6.14E-17 (1)	6.14E-17 (1)	4.46E-15 (1)	2.33E-09 (1)
f11	4.54E-01 (0)	7.55E-01 (0)	8.42E-01 (0)	2.58E-01 (0)	9.48E-02 (0)	4.84E-01 (0)
f12	6.18E-01 (0)	6.09E-01 (0)	8.64E-01 (0)	1.25E-01 (0)	8.64E-01 (0)	8.64E-01 (0)
f13	8.64E-01 (0)	3.93E-01 (0)	6.09E-01 (0)	8.64E-01 (0)	2.16E-13 (1)	6.09E-01 (0)
f14	1.82E-01 (0)	6.33E-02 (0)	8.62E-03 (1)	8.80E-01 (0)	2.65E-02 (1)	1.21E-01 (0)
f15	1.21E-1 (0)	2.32E-01 (0)	1.850E-01 (0)	2.33E-01 (1)	2.32E-01 (1)	1.84E-01
f16	6.09E-01 (0)	1.07E-01 (0)	1.72E-01 (0)	4.95E-01 (0)	1.18E-03 (1)	8.64E-01 (0)
f17	3.21E-18 (1)	6.14E-17 (1)	3.21E-18 (1)	1.59E-14 (1)	3.21E-18 (1)	1.59E-14 (1)
f18	7.58E-09 (1)	3.21E-18 (1)	3.21E-18 (1)	2.61E-12 (1)	2.61E-12 (1)	1.05E-15 (1)
f19	2.71E-10 (1)	3.21E-18 (1)	1.59E-14 (1)	2.16E-13 (1)	3.21E-18 (1)	6.14E-17 (1)
f20	2.68E-03 (1)	4.64E-08 (1)	3.21E-18 (1)	4.14E-05 (1)	1.05E-14 (1)	3.21E-18 (1)
f21	1.57E-12 (1)	7.48E-14 (1)	4.45E-14 (1)	2.98E-09 (1)	5.55E-14 (1)	1.05E-15 (1)
f22	2.16E-13 (1)	1.78E-08 (1)	3.21E-18 (1)	3.21E-18 (1)	2.82E-11 (1)	1.59E-14 (1)
f26	5.49E-01 (0)	1.31E-04 (1)	2.37E-05 (1)	1.81E-05 (1)	6.72E-07 (1)	3.70E-08 (1)
f27	6.47E-01 (0)	2.04E-02 (1)	3.58E-05 (1)	3.21E-18 (1)	2.84E-02 (1)	1.71E-15 (1)

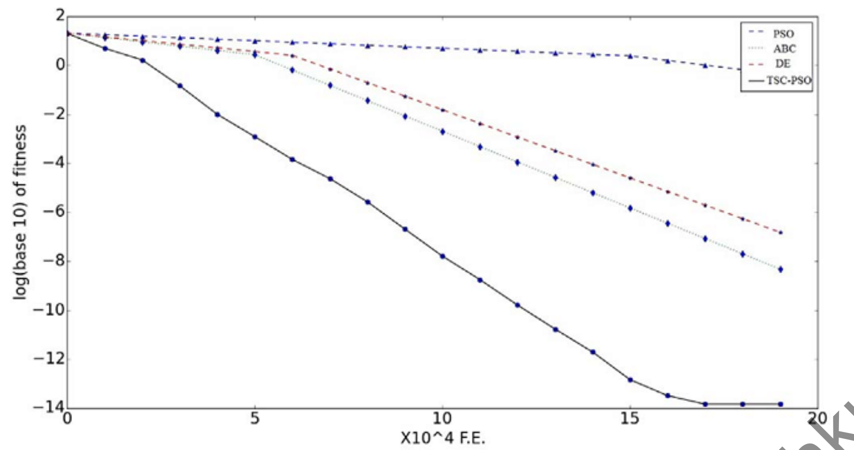
Comparison with Latest Best Algorithms

	TSC − PSO Mean	St. D	*L − SHADE* Mean	St. D	*CETMS* Mean	St. D	*FWA − DM* Mean	St. D
f1	1.38E+05	4.20E+04	1.70E+05	5.70E+04	0.00E+00	0.00E+00	2.28E+08	4.08E+07
f2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.62E+04	1.81E+04
f3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.95E+04	3.64E+03
f4	2.21E+02	3.30E+01	1.70E+02	3.10E+01	7.13E+01	4.43E+01	1.83E+02	1.00E+02
f5	2.00E+01	1.20E−06	2.10E+01	3.10E−02	2.01E+01	3.35E−01	2.10E+01	2.44E−02
f6	5.12E+00	1.65E+00	8.70E+00	2.30E+00	4.23E+00	2.60E+00	1.14E+02	2.54E+00
f7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.59E−01	1.99E+00	1.29E−01	3.14E−02
f8	0.00E+00	0.00E+00	1.10E−02	7.40E−03	2.53E+01	5.52E+00	1.08E4−02	5.40E+00
f9	5.76E+01	2.56E+00	3.40E+01	5.00E+00	2.45E+01	8.14E+00	5.52E+02	4.44E+01
f10	4.22E+01	1.12E+01	2.60E+01	5.80E+00	3.91E+03	1.26E+03	5.67E+03	3.04E+02
f11	2.12E+04	1.32E+03	1.10E+04	5.60E+02	4.29E+03	1.58E+03	1.46E4−04	6.30E+02
f12	1.96E−02	3.22E−03	4.40E−01	4.70E−02	1.51E−03	1.10E−03	1.21E+00	7.78E−02
f13	4.23E−01	2.45E−02	2.40E−01	2.10E−02	1.11E−01	4.21E−02	5.61E−01	3.61E−02
f14	2.66E−01	1.23E−02	1.20E−01	7.30E−03	5.00E−01	6.68E−02	1.89E−01	2.06E−02
f15	1.45E+03	2.30E+02	1.60E+01	1.20E+00	1.00E+01	2.24E+01	8.74E+01	5.87E+00
f16	3.86E+01	2.63E−01	3.90E+01	4.80E−01	2.28E+01	1.51E+00	4.35E+01	3.75E−01
f17	7.07E+03	1.59E+03	4.40E+03	7.10E+02	2.16E+03	5.40E+02	2.31E+07	5.63E+06
f18	4.14E+02	7.86E+01	2.20E+02	1.70E+01	2.65E+02	6.47E+01	5.68E+03	8.70E+03
f19	6.41E+01	1.21E−01	9.60E−01	2.30E+00	1.21E+01	3.14E+00	6.34E+01	2.43E+00
f20	9.86E+01	3.35E+01	1.50E+02	5.20E+01	3.51E+02	1.53E+02	6.93E+04	1.10E+04
f21	3.12E+03	6.54E+02	2.30E+03	5.30E+02	1.36E+03	4.22E+02	9.57E+06	2.31E+06
f22	8.77E+02	1.54E+02	1.10E+03	1.90E+02	3.68E+02	1.90E+02	1.51E+03	1.34E+02
f23	3.00E+02	6.40E−11	3.50E+02	2.80E−13	3.44E+02	2.87E−13	3.46E+02	2.18E−01
f24	3.05E+02	4.12E+00	3.90E+02	2.90E+00	2.75E+02	1.14E+00	3.63E+02	2.85E+00
f25	2.00E+02	3.40E−12	2.00E+02	4.00E−13	2.08E+02	2.77E+00	3.03E+02	1.74E+01
f26	1.06E+02	6.60E−13	2.00E+02	6.20E−13	1.75E+02	1.02E+02	1.61E+02	5.88E+0 I
f27	4.00E+02	2.20E−05	3.80E+02	3.30E+01	4.54E+02	7.34E+01	3.14E+03	4.13E+02
f28	3.00E+02	4.51E−05	2.30E+03	4.60E+01	1.27E+03	8.63E+01	1.60E+03	3.42E+02
f29	7.33E+02	1.21E+01	8.00E+02	7.60E+01	2.26E+06	9.16E+06	2.70E+02	5.23E+00
f30	2.12E+03	5.46E+02	8.30E+03	9.60E+02	9.81E+03	1.73E+03	2.23E+03	1.16E+03

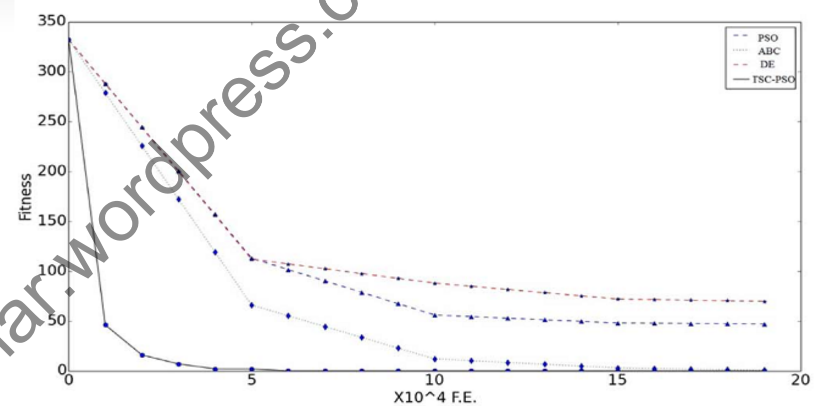
SW and Wilcoxon Test

	TSC − PSO SW: p (h)	*L − SHADE* SW: p (h)	W: p (h)	*CETMS* SW: p (h)	W: p (h)	*FWA − DM* SW: p (h)	W: p (h)
f1	3.56E−03(1)	1.18E−02(1)	4.13E−03(1)	7.51E−02(0)	7.15E−07(1)	7.01E−02(0)	1.69E−06(1)
f2	8.77E−02(0)	1.27E−01(0)	1.51E−01(0)	6.14E−02(0)	8.80E−02(0)	2.31E−03(1)	7.45E−07(1)
f3	5.45E−02(0)	5.45E−02(0)	7.85E−01(0)	5.15E−02(0)	1.07E−01(0)	5.36E−03(1)	3.77E−07(1)
f4	4.60E−02(1)	7.47E−03(1)	1.15E−09(1)	8.19E−03(1)	2.39E−06(1)	4.20E−03(1)	1.40E−01(0)
f5	1.18E−06(1)	2.85E−03(1)	1.88E−06(1)	2.48E−03(1)	1.05E−02(1)	1.55E−03(1)	2.78E−06(1)
f6	5.57E−02(0)	1.39E−02(1)	1.08E−10(1)	1.82E−01(0)	5.96E−02(0)	4.47E−02(1)	2.80E−06(1)
f7	7.44E−02(0)	7.12E−02(0)	8.78E−02(0)	1.39E−02(1)	2.65E−02(1)	1.20E−02(1)	2.55E−06(1)
f8	5.12E−02(0)	2.03E−02(1)	2.16E−13(1)	1.16E−02(1)	1.34E−07(1)	4.09E−02(1)	2.75E−07(1)
f9	1.40E−02(1)	2.42E−01(0)	1.28E−06(1)	1.95E−02(1)	2.12E−06(1)	3.55E−03(1)	2.62E−06(1)
f10	9.15E−04(1)	2.62E−03(1)	3.64E−12(1)	6.34E−02(0)	1.63E−06(1)	2.21E−02(1)	3.28E−06(1)
f11	5.07E−03(1)	1.84E−02(1)	2.87E−06(1)	5.71E−02(0)	3.87E−06(1)	8.45E−03(1)	2.13E−06(1)
f12	4.82E−02(1)	1.79E−02(1)	1.30E−06(1)	7.07E−03(1)	4.23E−07(1)	6.09E−02(0)	2.44E−06(1)
f13	2.73E−03(1)	8.52E−03(1)	3.12E−06(1)	4.24E−03(1)	1.69E−06(1)	2.65E−02(1)	3.63E−07(1)
f14	2.05E−02(1)	1.56E−02(1)	1.07E−06(1)	4.04E−03(1)	6.15E−07(1)	1.76E−01(0)	1.12E−06(1)
f15	2.13E−01(0)	8.12E−03(1)	1.76E−06(1)	4.68E−02(1)	2.11E−06(1)	1.23E−02(1)	1.83E−06(1)
f16	1.32E−01(0)	4.75E−03(1)	5.22E−05(1)	2.24E−02(1)	3.50E−06(1)	2.58E−03(1)	2.07E−06(1)
f17	4.39E−02(1)	1.14E−02(1)	2.77E−15(1)	2.18E−02(1)	3.77E−06(1)	4.76E−02(1)	2.55E−06(1)
f18	1.09E−02(1)	3.30E−03(1)	3.83E−06(1)	1.65E−02(1)	4.33E−13(1)	1.16E−01(0)	8.66E−05(1)
f19	4.66E−03(1)	7.57E−04(1)	9.63E−07(1)	1.50E−02(1)	2.70E−06(1)	7.52E−02(0)	3.37E−01(0)
f20	1.26E−02(1)	1.16E−02(1)	1.28E−06(1)	2.28E−02(1)	6.51E−15(1)	2.42E−03(1)	1.16E−06(1)
f21	2.56E−02(1)	4.76E−02(1)	3.70E−08(1)	1.70E−02(1)	2.69E−06(1)	6.63E−04(1)	2.78E−06(1)
f22	1.03E−02(1)	2.29E−03(1)	1.31E−07(1)	1.46E−03(1)	9.76E−18(1)	4.12E−04(1)	2.72E−07(1)
f23	1.22E−01(0)	5.87E−02(0)	1.02E−06(1)	1.71E−01(0)	8.96E−07(1)	1.07E−02(1)	2.67E−06(1)
f24	2.47E−02(1)	1.13E−03(1)	3.38E−06(1)	1.87E−02(1)	1.38E−06(1)	6.28E−02(0)	3.12E−06(1)
f25	5.43E−02(0)	2.11E−01(0)	8.15E−01(0)	2.10E−02(1)	2.70E−06(1)	2.50E−03(1)	2.69E−08(1)
f26	1.44E−01(0)	1.24E−01(1)	2.41E−06(1)	5.16E−03(1)	8.66E−05(1)	1.24E−01(0)	7.45E−07(1)
f27	9.46E−07(1)	7.14E−03(1)	3.72E−03(1)	4.75E−03(1)	8.66E−05(1)	7.30E−03(1)	1.55E−06(1)
f28	2.18E−05(1)	1.59E−01(0)	3.66E−06(1)	1.69E−02(1)	4.60E−09(1)	9.52E−02(0)	1.85E−06(1)
f29	5.03E−03(1)	4.69E−02(1)	5.78E−06(1)	2.42E−01(0)	2.32E−01(0)	3.47E−02(1)	1.70E−06(1)
f30	9.84E−04(1)	1.14E−01(0)	1.84E−06(1)	1.21E−02(1)	3.08E−06(1)	3.19E−03(1)	6.42E−01(0)

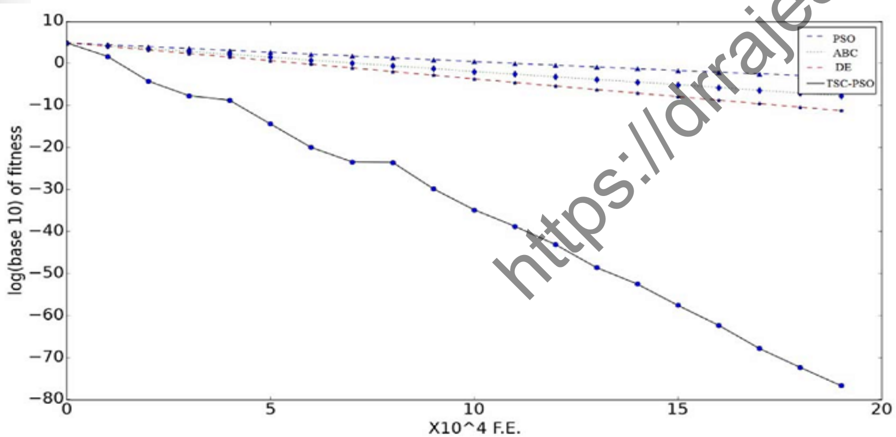
Convergence Plots



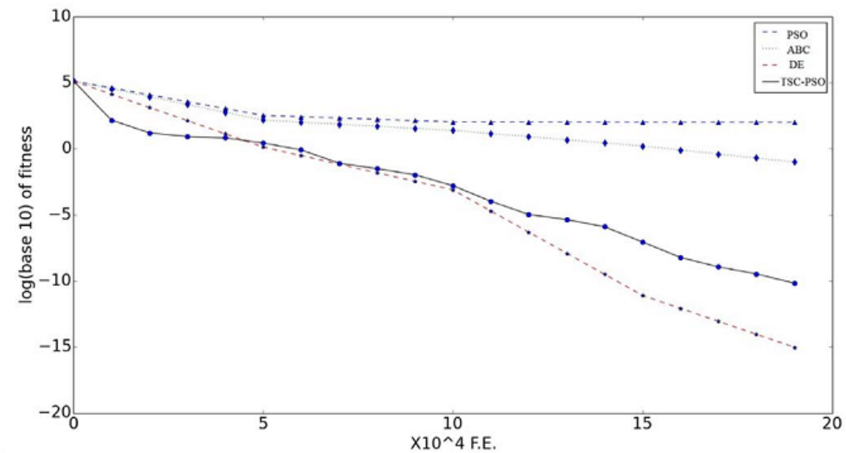
(a) Ackley function



(a) rastrigin function



(b) Elliptic function



(b) Rosenbrock function

Thank You