Development of Directed Bee Colony Optimization Algorithm

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Goal of Optimization

Find values of the variables that minimize or maximize the objective function while satisfying the constraints.

Mathematical Formulation of Optimization Problems

minimize the objective function

$$\min\ f(x), x = \left(x_1, x_2, \ldots\ldots, x_n\right)$$

subject to constraints

$$c_i(x) \geq 0$$

$$c_i\left(x\right) = 0$$

Example

$$\min\left[\left(x_1 - 2\right)^2 + \left(x_2 - 1\right)^2\right]$$

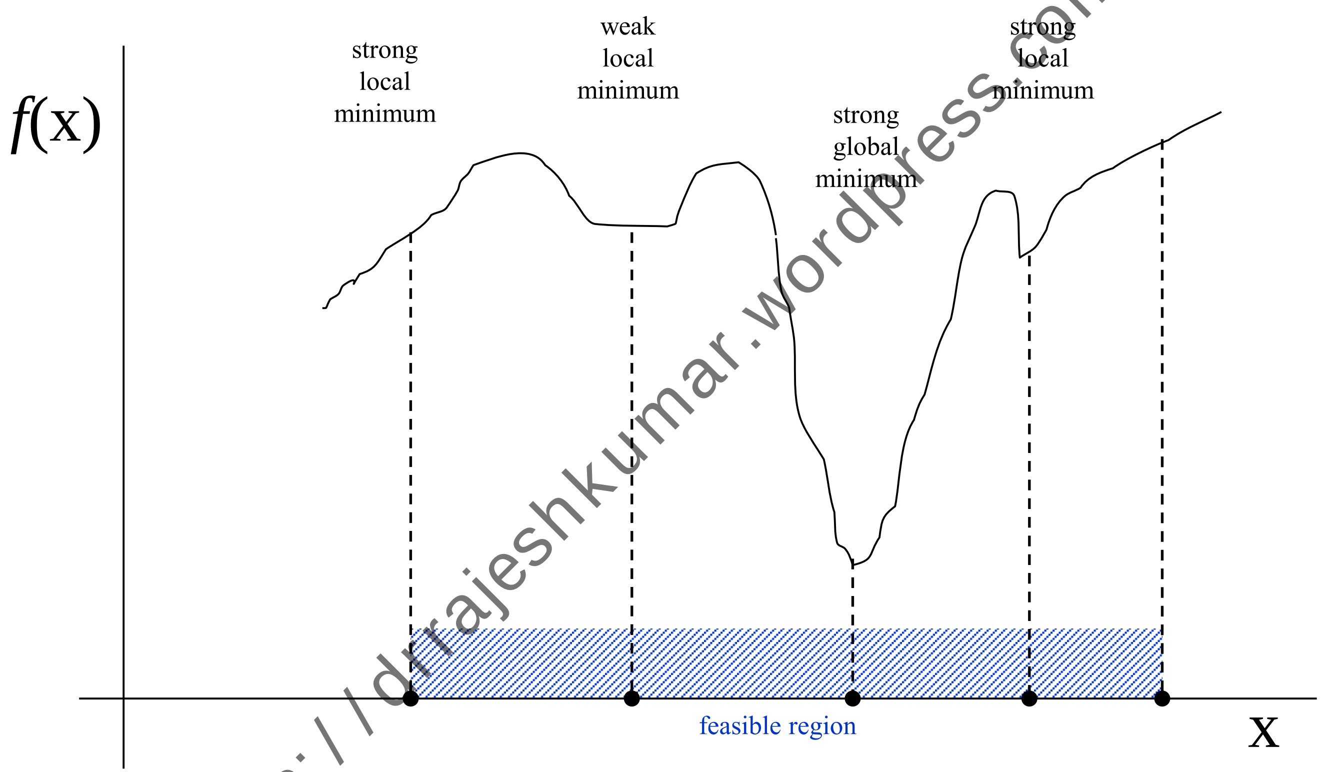
$$\text{subject}: x_1^{\ 2} - x_2^{\ 2} \leq 0$$

$$x_1 + x_2 \leq 2$$

Classification of Optimization Problems

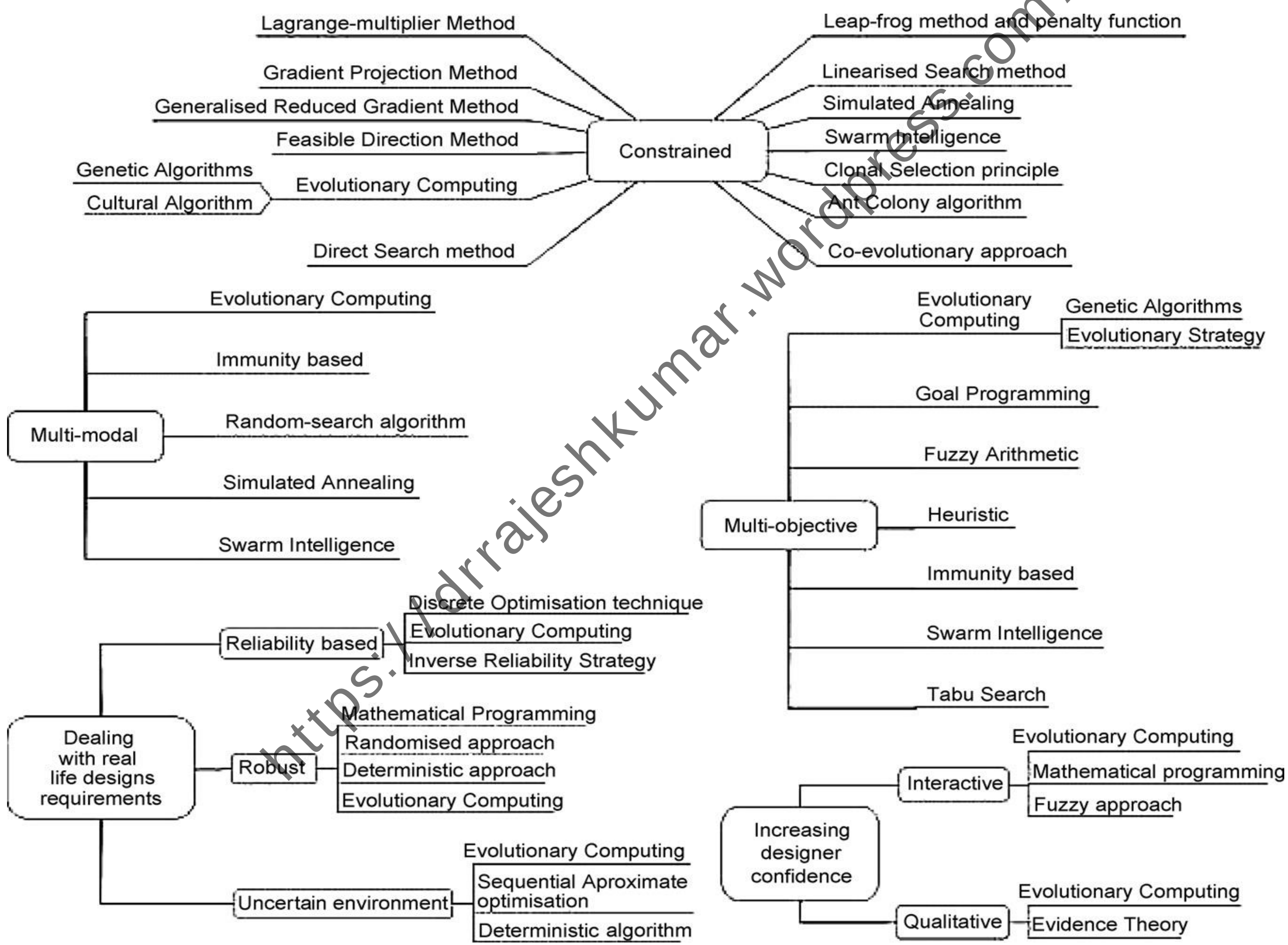
- Single variable
- Multi-variable
- Constrained
- Non-constrained
- Single objective
- Multi-objective
- Linear
- Non-linear

Local and Global Optimizers



- which of the minima is found depends on the starting point
- such minima often occur in real applications

Major Techniques used in Optimization



Motivation

- The new optimization algorithm inspired by group decision-making process of honey bees for best nest site which is analogous to the optimization process.

Working principle of bee in nature

- Initially some bees go for searching the nectar sources.
- Then the bee loads the nectar and returns to the hive to relinquish and performs the waggle dance.
- The waggle dance gives information about the quality, distance and direction of flower patch.
- The recruited bees will evaluate the quality of source upon exploring it.
- If the patch is still good enough for nectar, then they perform waggle dance to convey the same which enables quiescent bees in recruitment of more bees to explore the patch.

Decision Process

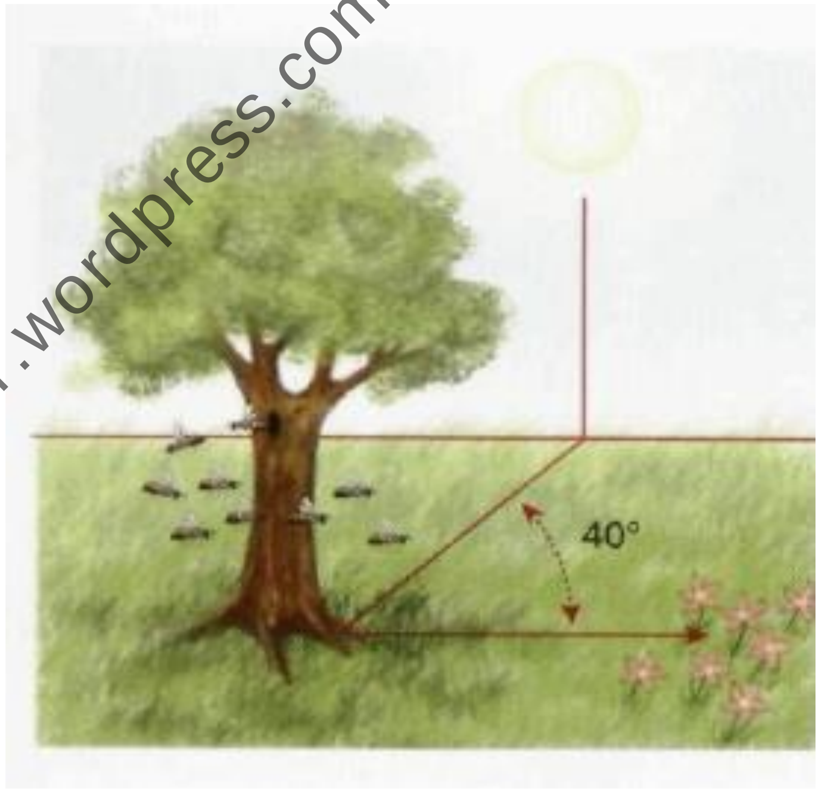
- Two methods used by bee swarms to reach to a decision for finding out the best nest site.
- Consensus: Widespread agreement among the group is taken into account.
- Quorum: The decision for best site happens when a site crosses the quorum (threshold) value.
- In DBC, both consensus and quorum have been mimicked, compared and presented for finding out the optimum solution.

Directed Bee Colony Algorithm

- 1/10 th of total bees (known as scouts) go for searching next site.
- Information of best site through waggle dance
- Decision making Process:
  – Consensus
  – Quorum
- Inform other bees through piping

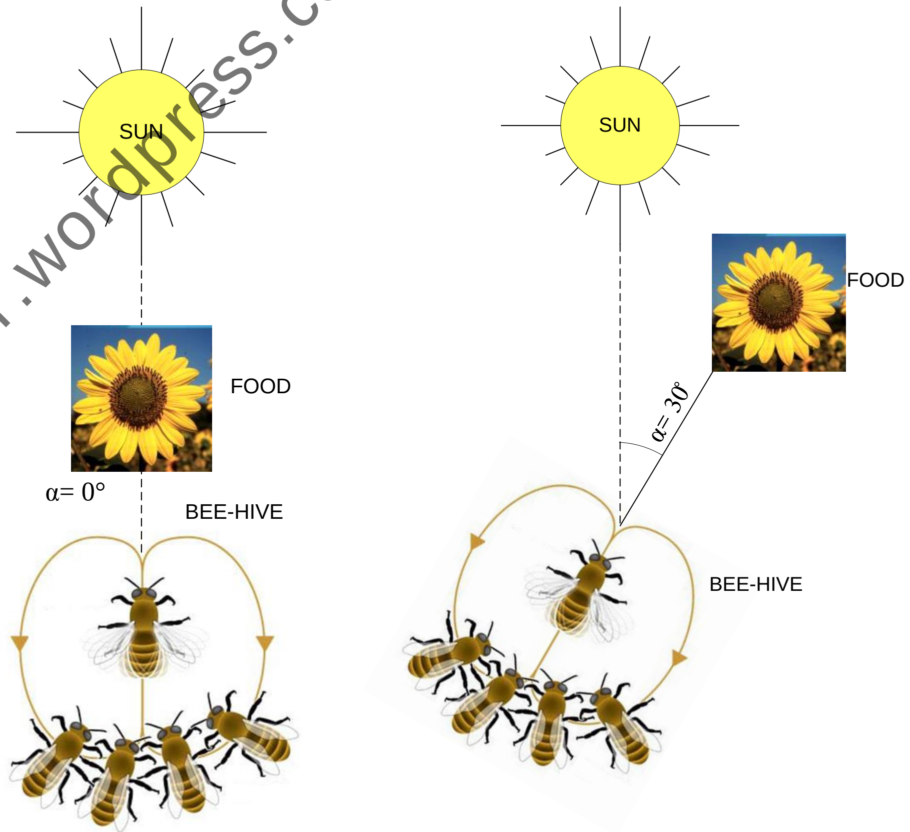
Honey Bee Algorithm

- Waggle dance
- Calculation of angle b/w sun and flowers
- waggle dance occurs on the surface of a swarm rather than on the combs inside a hive.



Honey Bee Algorithm

- Bee runs through a figure-eight pattern on a vertical comb
- Angle of the run indicates the direction to the food source
- Duration of the waggle run relates to the distance to the food source



The proposed Directed Bee Colony Optimization algorithm

- **DBC is a computational system in which several bees work, interact with each other and in unison take decision to achieve goals.**
- **Assumptions made in the proposed algorithm:**
  - Bees live and act in a given environment.
  - Bees attempt to achieve particular goals or perform particular tasks.
  - Information exchange process accompanies no losses.
  - Bees will never die and hence number of bees remains constant during the process.

DBC Algorithm

- **In DBC, all bees live in an environment i.e. FSR. An environment is organized in a structure as shown below for d(1,2,3) parameters:**

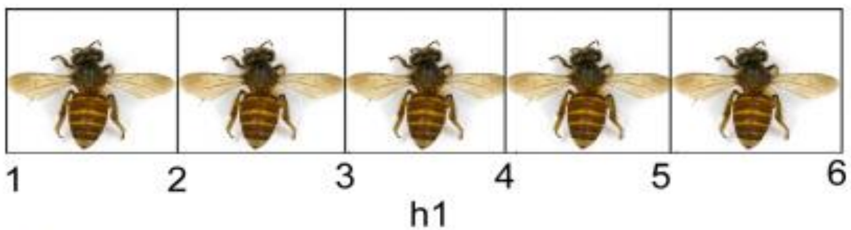


Fig. 1. Domain of the objective function with one independent parameter.

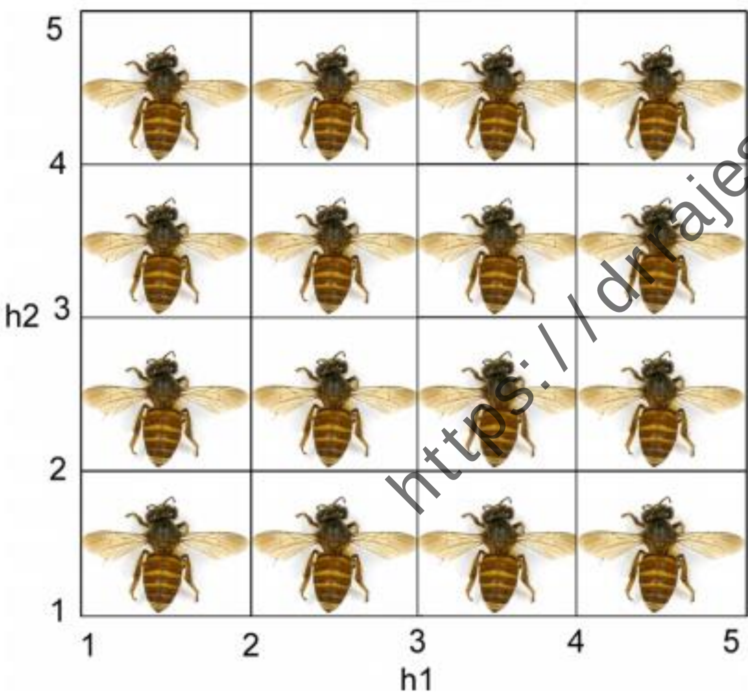


Fig. 2. Domain of the objective function with two independent parameters.

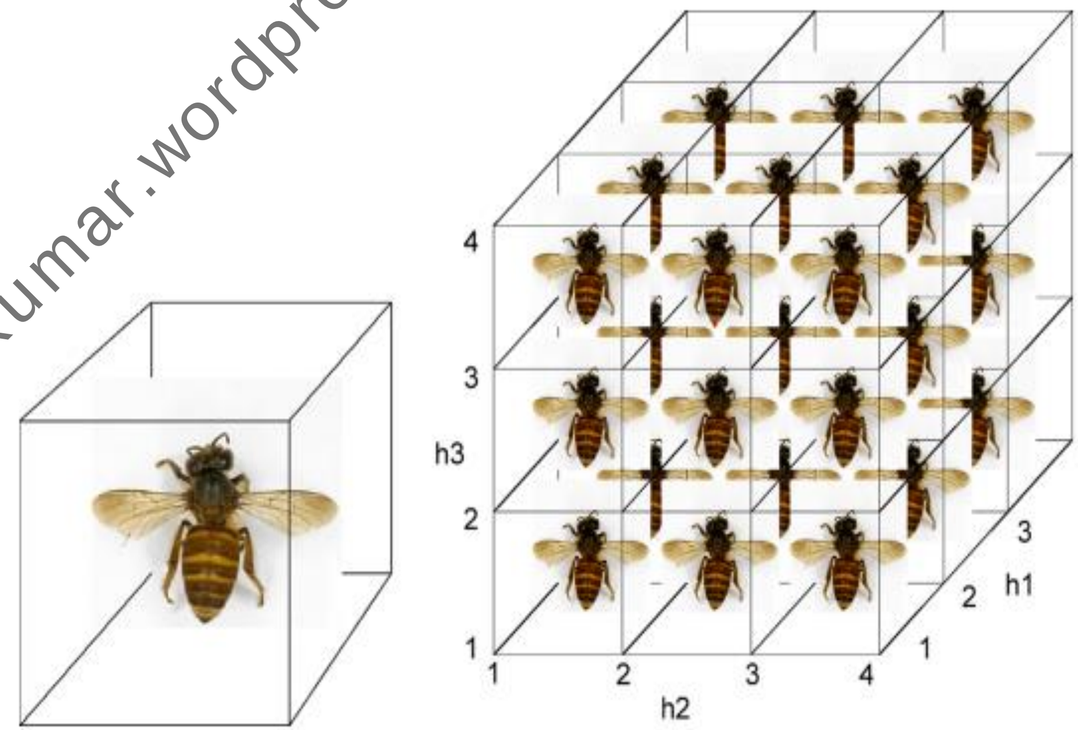


Fig. 3. Domain of the objective function with three independent parameters.

- **A point inside each volume is chosen as the starting point for the search process, which in proposed approach is the midpoint of that volume through various investigations.**
- **The midpoint of total cluster can be calculated using (1)**

$$\left[\frac{W_{1i}+W_{1f}}{2}, \frac{W_{2i}+W_{2f}}{2}, \ldots\ldots, \frac{W_{di}+W_{df}}{2}\right] \quad (1)$$

- **If there is one parameter only one bee explores the search.**
- **If the parameters are $W_{1i} = 1, W_{1f} = 6$ then 5 bees are sent for exploration for the axis given by two parameters $d_1 and\ d_2$**
- **If the parameters are $W_{1i} = 1, W_{1f} = 5\ and\ W_{2i} = 1, W_{2f} = 5$ then bees are sent for exploration.**

Bee search methodology

- **In the proposed optimization, the analogy has been derived with the search approach adopted by bees and the tendency of remembering only four locations at a time.**

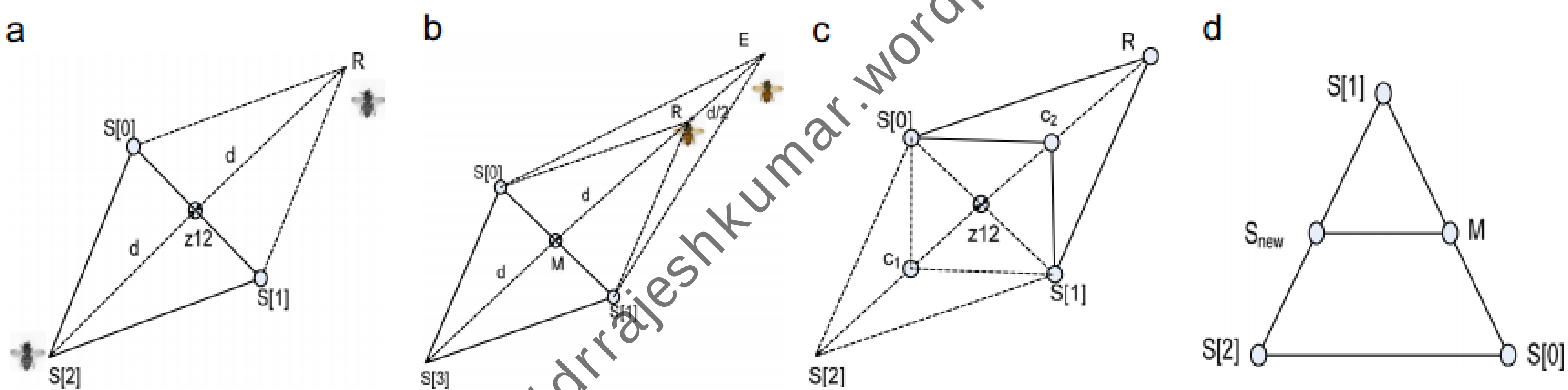
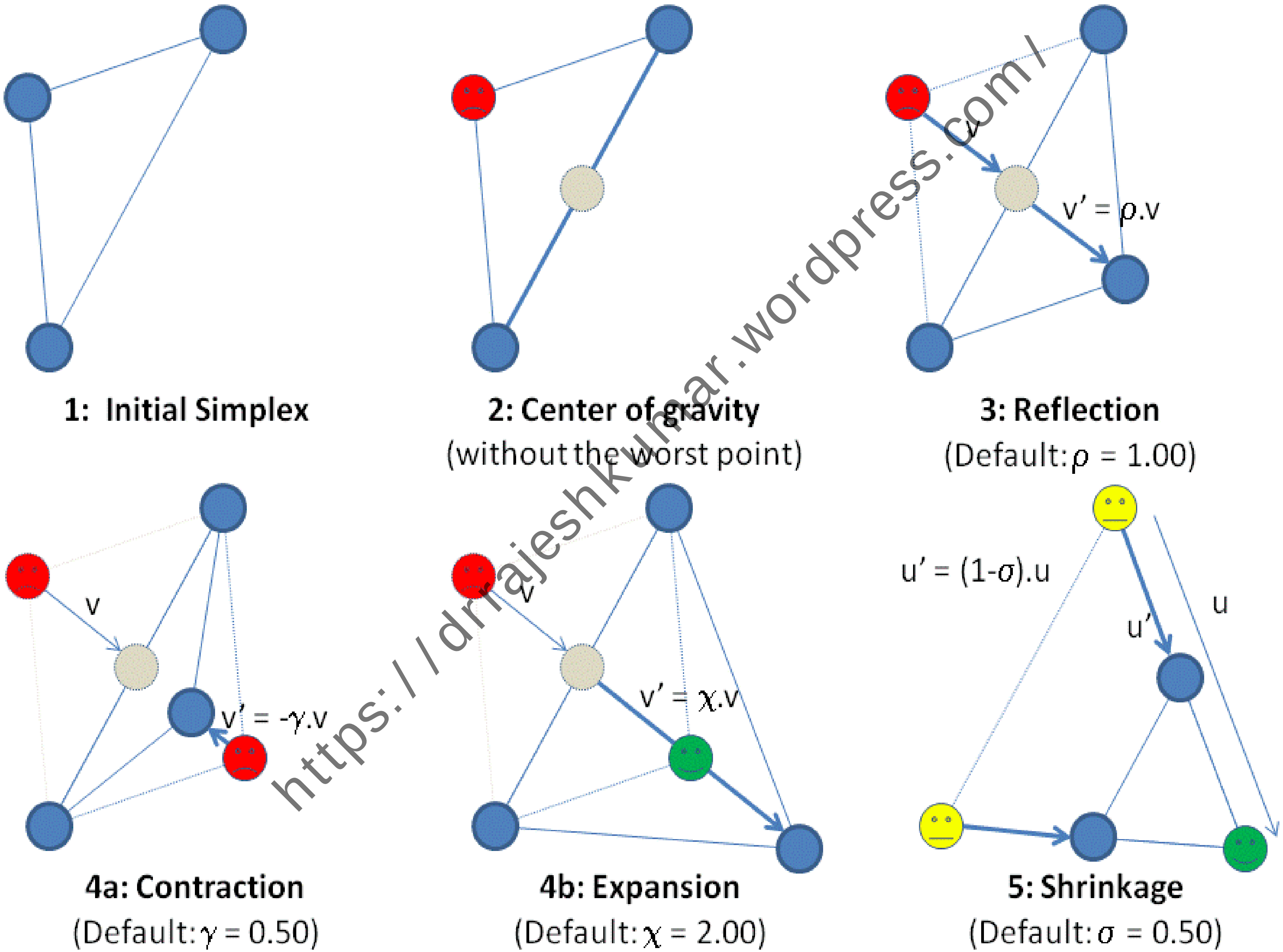
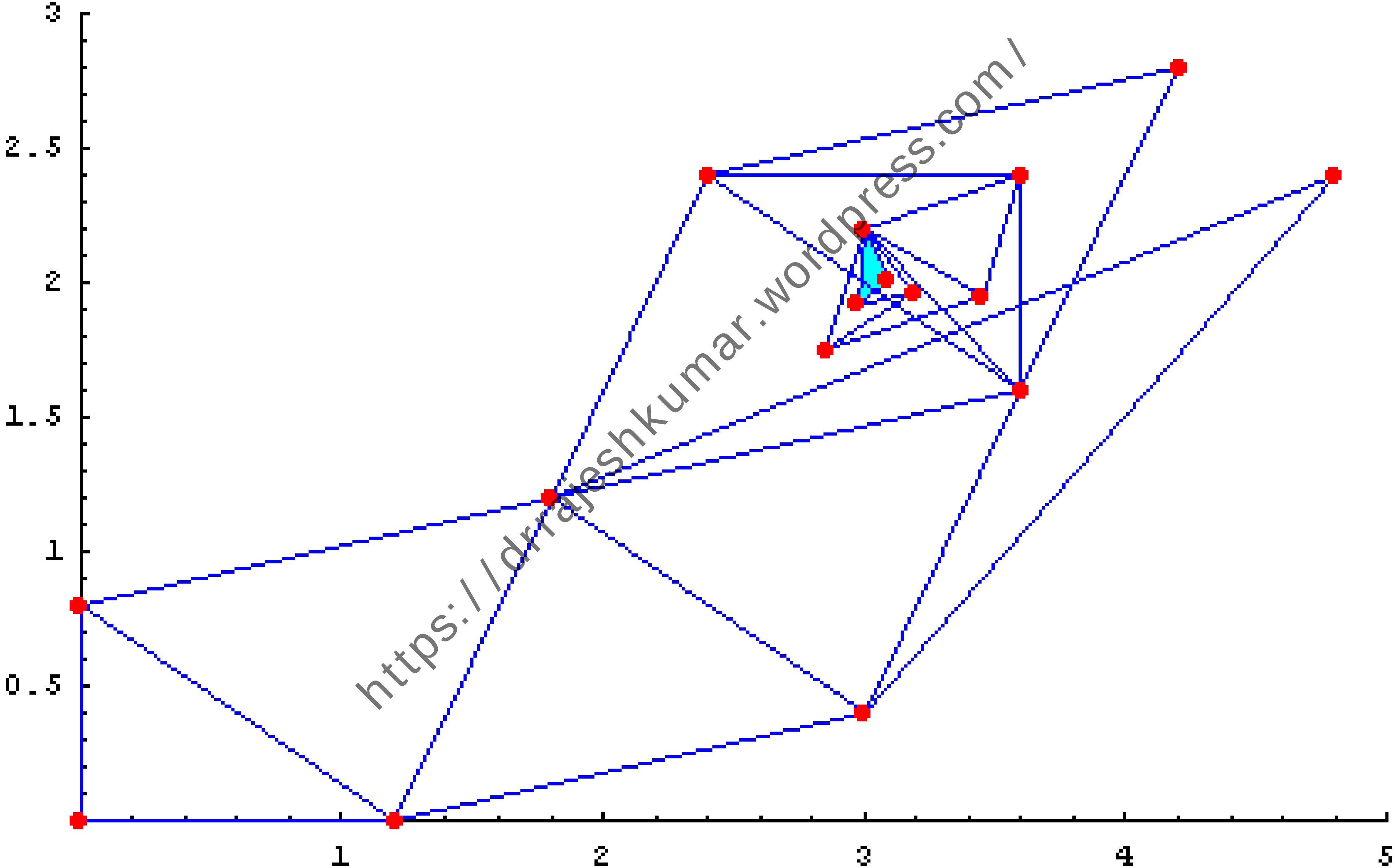


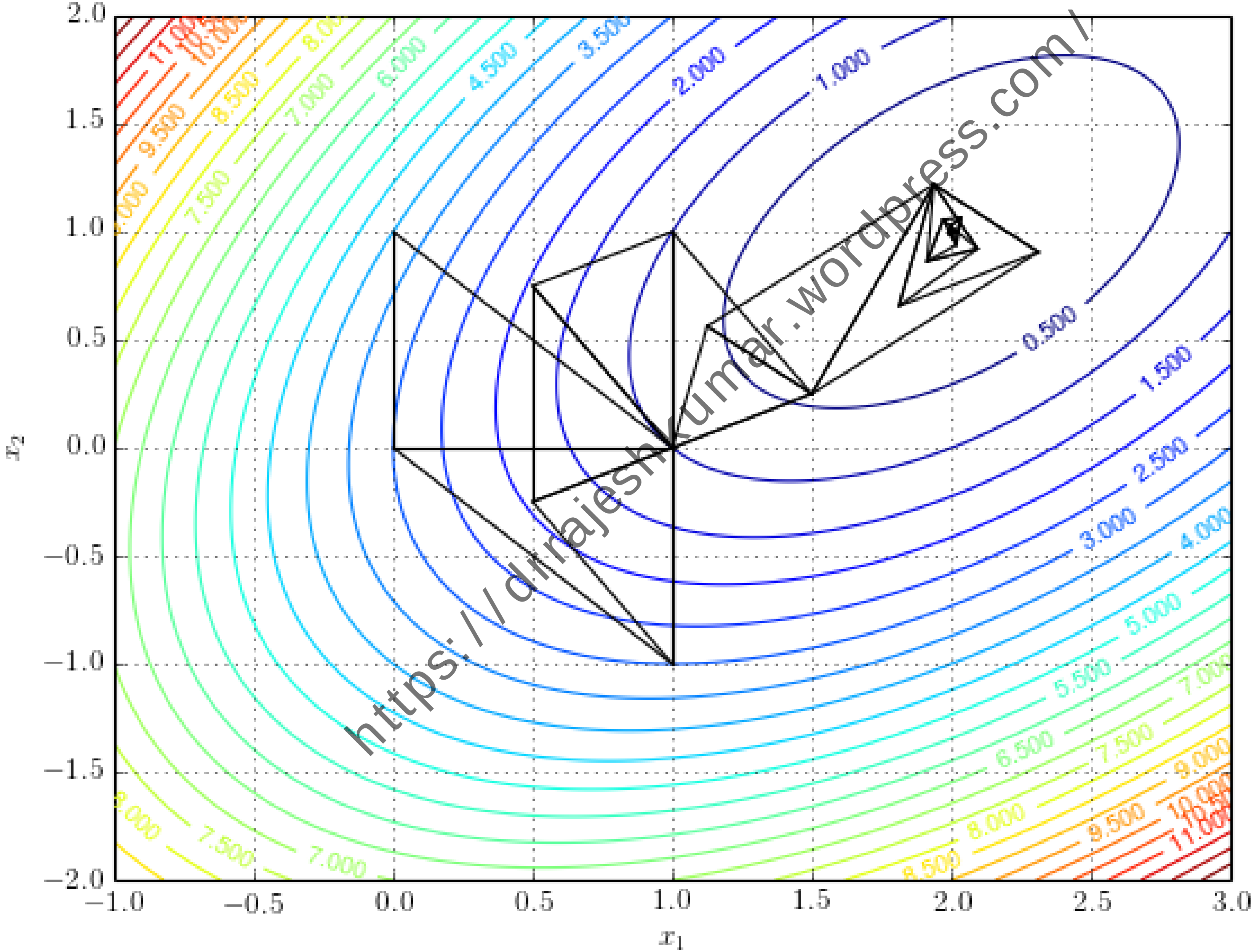
Fig. 4. Bees search movements with the proposed optimization algorithm: (a) starting of the motion in search of solution, (b) extension in the direction of good optimal point, (c) contraction of the movement in case optimal point quality is not good, and (d) shrinking of the space towards optimistic solution.

- **The author has used a very popular optimization technique (Fig. 4) usually known as Nelder–Mead method (NM) based on geometric operations (reflection, expansion, contraction and shrinking).**

Main steps of the simplex algorithm




3
2.5
2
1.5
1
0.5
1
2
3
4
5


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
x_2
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x_1
0.500
1.000
1.500
2.000
2.500
3.000
3.500
4.000
4.500
5.000
5.500
6.000
6.500
7.000
7.500
8.000
8.500
9.500
10.000
11.000

Communication of the information through waggle dance

- **The individual optimal solutions of bees are communicated to the centralized system that chooses best preferable solution.**
- **Definition 5. A food quality index FQI : FSR- $\mathcal{R}$ is a measure of the quality of the solution that is represented by the bee food search procedure.**
- **For optimal minimum cases it selects the best optimal solution which can mathematically expressed as**

$$\mathrm{FQI}(i) = \min(\mathrm{FV}(i)) \tag{2}$$

where, FV(i) represent the different search value equivalent to food value obtained by a bee.

Global Optimal Solution Selection

- **When bee encounters multiple global optimal solutions with same fitness value it selects the optimal solution nearer to the starting point (Fig. 5).**
- **This is due to the natural phenomenon used by bees when it find nectar site with same fitness value.**

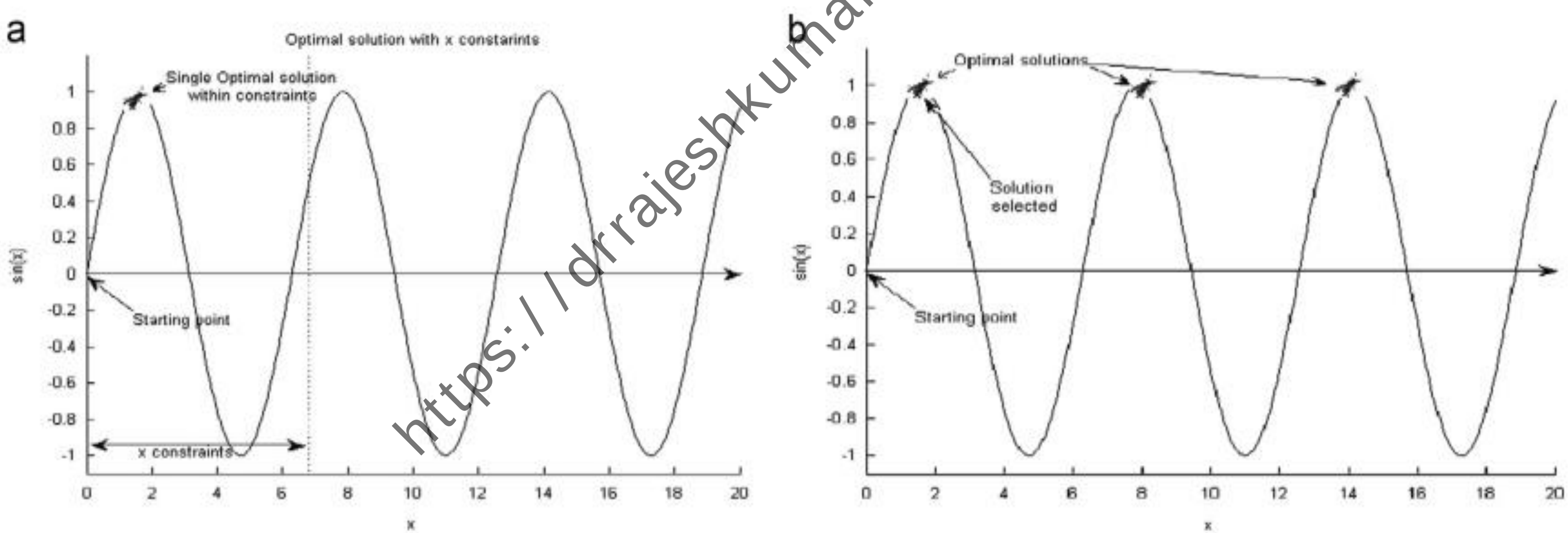


Fig. 5. Solution report by bee to hive: (a) with *x* constraints and (b) without constraints.

Bee based decision processes-consensus and quorum

- *Consensus:* **Once exploration and waggle dance (transmission of data) is finished the global optimized point is chosen by comparing the Fitness Values of all the optimized points in the optimum vector table i.e. global best, gbest as in case of PSO [10,11].**
- **Point with the lowest Fitness Value is selected as the global optimized point (X_G).**

- *Quorum:* **In quorum method, if the number of bees providing an optimized solution reaches quorum threshold (εq) then that optimum solution is considered as the final solution and further search of optimum result is stopped and hence saves time.**

Flow Chart for DBC

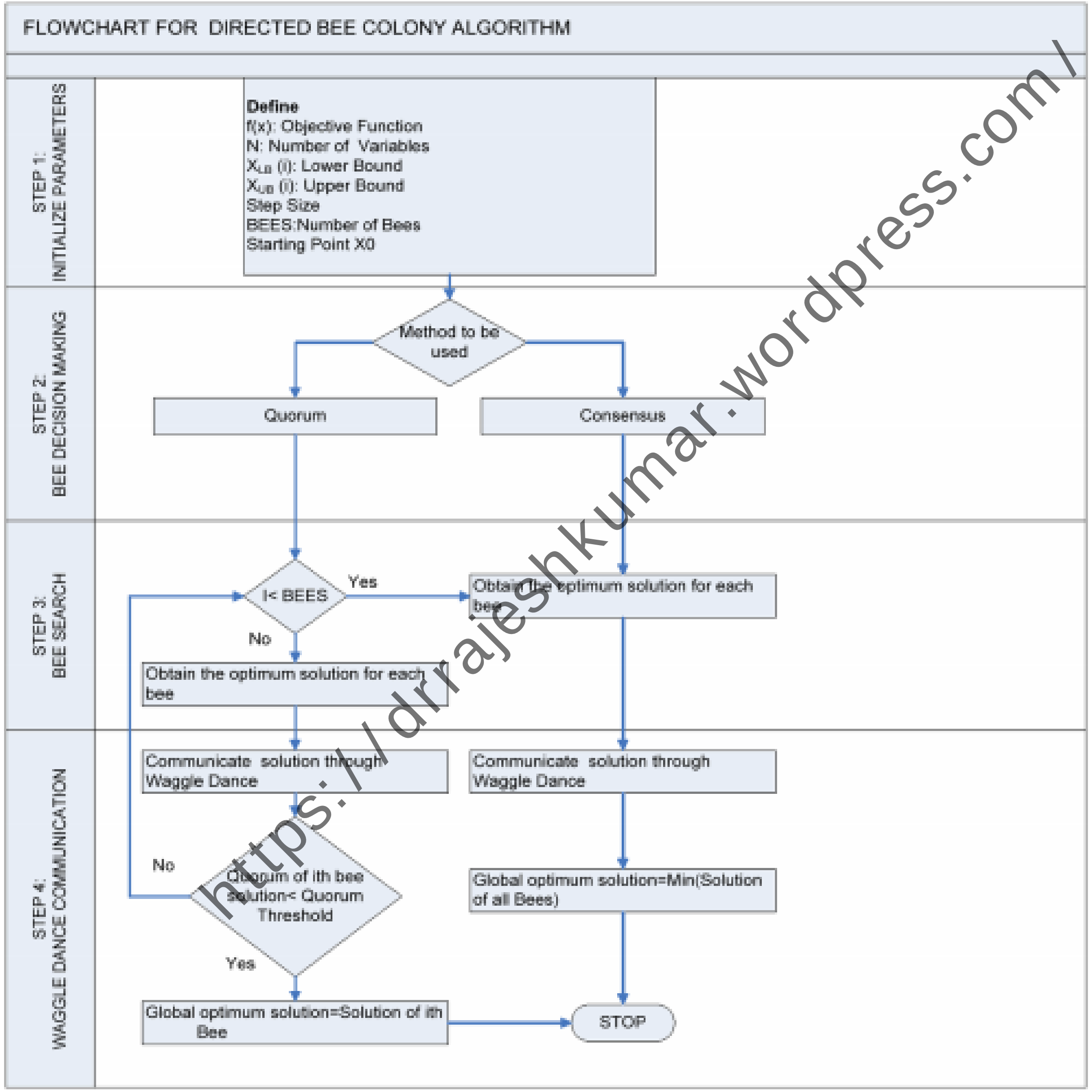


Fig. 6. Flowchart for the proposed DBC algorithm.

Table 1
Benchmark problems.

Function	Function range	$(\vec{x}^*)$	f^*_{min}
Sphere $f_1(\vec{x}) = \sum_{i=1}^{N} x_i^2$	$-100 \le x_i \le 100$	0	0
Schwefel's Problem 2.22 $f_2(\vec{x}) = \sum_{i=1}^{N} \lvert x_i \rvert + \prod_{i=1}^{N} \lvert x_i \rvert$	$-10 \le x_i \le 10$	0	0
Rosenbrock $f_3(\vec{x}) = \sum_{i=1}^{N} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2$	$-30 \le x_i \le 30$	(1, 1, …, 1)	0
Rotated hyper−ellipsoid $f_4(\vec{x}) = \sum_{i=1}^{N} \left(\sum_{j=1}^{i} x_j\right)^2$	$-100 \le x_i \le 100$	0	0
Generalized Swefel's Problem 2.26 $f_5(\vec{x}) = -\sum_{i=1}^{N} (x_i \ \sin(\sqrt{\lvert x_i \rvert})$	$-512 \le x_i \le 512$	(420.9687, …, 420.9687)	$-418.982887 \times N$
Rastrigin $f_6(\vec{x}) = \sum_{i=1}^{N} (x_i^2 - 10 \ \cos(2\pi x_i) + 10)$	$-5.12 \le x_i \le 5.12$	0	0
Ackley $f_7(\vec{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{N}\sum_{i=1}^{N} \ \cos(2\pi x_i)}\right) - \exp(\frac{1}{N}\sum_{i=1}^{N} \ \cos(2\pi x_i)) + 20 + e$	$-32 \le x_i \le 32$	0	0
Griewank $f_8(\vec{x}) = \frac{1}{4000}\sum_{i=1}^{N} x_i^2 - \prod_{i=1}^{N} \ \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$-600 \le x_i \le 600$	0	0

Table 2
Shifted and shifted rotated benchmark problems.

Function	Function range	$(\vec{x}^*)$	f^*_{min}
Shifted sphere (CEC2005 F1) $f_9(\vec{x}) = \sum_{i=1}^{N} z_i^2 + f_{bias_1}$ where $z = x - o$	$-100 \le x_i \le 100$	$o = [o(1), o(2) \ldots o(n)]$	$f_{bias_1} = -450$
Shifted Shwefel's Problem 1.22 (CEC2005 F2) $f_{10}(\vec{x}) = \sum_{i=1}^{N}\left(\sum_{j=1}^{i} z_j\right)^2 + f\_bias_2$ where $z = x - o$	$-10 \le x_i \le 10$	$o = [o(1), o(2) \ldots o(n)]$	$f_{bias_2} = -450$
Shifted Rosenbrock (CEC2005 F6) $f_{11}(\vec{x}) = \sum_{i=1}^{N} (100(z_{i+1} - z_i^2)^2 + (z_i - 1)^2 + f\_bias_6$ where $z = x - o + 1$	$-30 \le x_i \le 30$	$o = [o(1), o(2) \ldots o(n)]$	$f_{bias_6} = 390$
Shifted Rastrigin (CEC 2005 F9) $f_{12}(\vec{x}) = \sum_{i=1}^{N} (z_i^2 - 10 \ \cos(2\pi z_i) + 10) + f\_bias_9$ where $z = x - o$	$-5 \le x_i \le 5$	$o = [o(1), o(2) \ldots o(n)]$	$f\_bias_9 = 330$
Shifted rotated high conditional elliptic (CEC 2005 F3) $f_{13}(\vec{x}) = \sum_{i=1}^{N} (10^6)^{(i-1)/(N-1)} \cdot (z_i)^2 + f\_bias_3$ where $z = (x - o)M$; M is the linear transformation matrix	$-100 \le x_i \le 100$	$o = [o(1), o(2) \ldots o(n)]$	$f_{bias_3} = -450$
Shifted rotated Griewank's function without bounds (CEC 2005 F7) $f_{14}(\vec{x}) = \frac{1}{4000}\sum_{i=1}^{N} z_i^2 - \prod_{i=1}^{N} \ \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f\_bias_7$ where $z = (x - o)M$; M is the linear transformation matrix	$-100 \le x_i \le 100$	$o = [o(1), o(2) \ldots o(n)]$	$f_{bias_7} = -180$
Rotated hybrid composition function CF1 $f_{15}(\vec{x})$ = marked as F16 in CEC2005 benchmark problem set	$-5 \le x_i \le 5$	–	$f_{bias_{16}} = 10$
Rotated hybrid composition function CF2 $f_{16}(\vec{x})$ = marked as F18 in CEC2005 benchmark problem set	$-5 \le x_i \le 5$	–	$f_{bias_{18}} = 10$

Impact of bee decision making on the performance of DBC

- **The decision making processes consensus and quorum are compared (Table 3).**
- **Performance can be increased by switching between the two decision making processes.**
- **Two choices of parameters can be used: number of bees and quorum (threshold) value.**
- **As the quorum value is increased the optimal solution approaches towards the global value at an expense of time (Fig. 7).**

Table 3
Experimental results for banana function for consensus and quorum.

	f_1	Quorum threshold	Time (s)
Consensus	0		1.3999102
Quorum DBC	1.2173e−009	10	0.1949080
	1.2173e−009	50	0.4097410
	1.2173e−009	60	0.4621518
	6.1001e−029	70	1.40418157
	0	80	1.39418157
	0	100	1.41418157

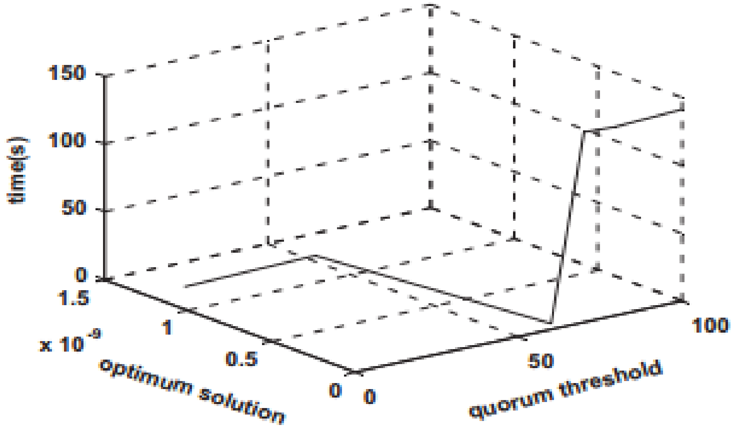


Fig. 7. Variation in solution of Rosenbrock banana function for quorum method.

Choice of parameters:

- **Effect of number of bees: More the number of bees, more is the computational time taken for processing but with increased accuracy towards the optimal result (Fig. 8).**

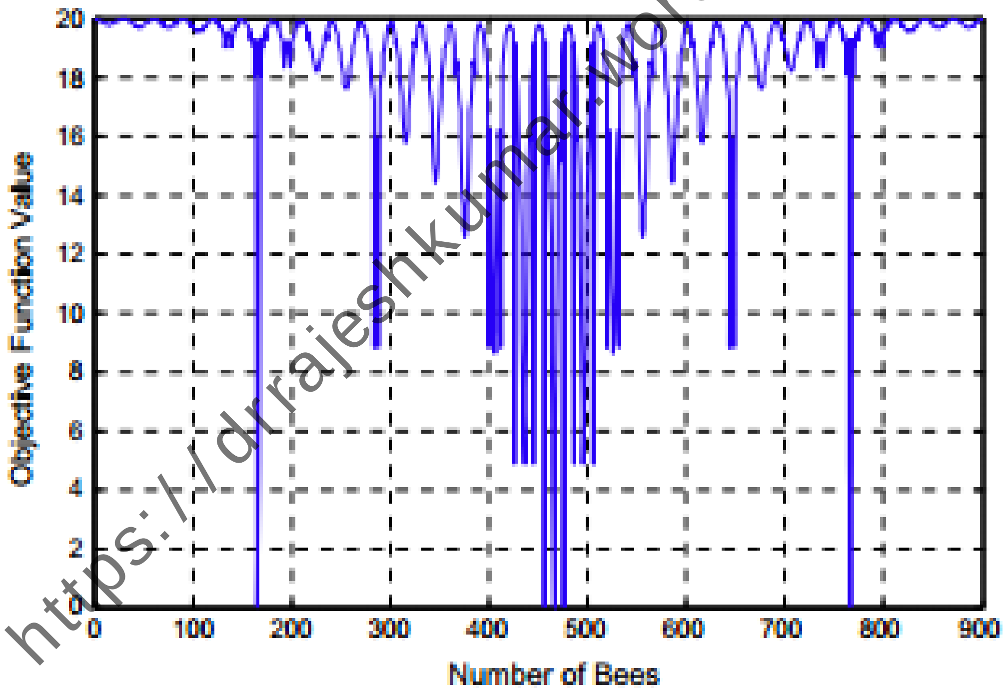


Fig. 8. Variation in Ackley function value as found by different bees.

Comparison with other similar algorithms: Case study-I: normal benchmark problems

Mean and standard deviation (SD) of the benchmark function optimization results ($N=2$).

Algorithm	Functions							
	Sphere	Schwefel's 2.22	Rosenbrock	Rotated-hyper	Schwefel's 2.26	Rastrigin	Ackley	Griewank
GA								
AE	1.87E−04	1.7152E−01	9.46128E−03	2.82441E−11	−8.3796E+02	1.3906E+00	1.13E+00	7.678E−03
SD	3.20E−05	7.2851E−02	2.1562E+00	5.4379E−11	8.1302E−08	8.2424E−01	4.070E−01	4.1207E−03
EA								
AE	5.26E−11	3.127E−06	4.325E−05	0	−8.3769E+02	2.04E−08	7.342E−06	1.23E−08
SD	8.25E−11	2.456E−06	3.654E−05	4.12E−12	4.65E−06	2.79E−08	5.63E−06	2.08E−08
PSO								
AE	8.307E−06	2.4000E−06	3.1881E−06	2.6506E−11	−8.3796E+02	3.6559E−06	6.306E−06	6.3295E−06
SD	4.390E−06	1.7082E−06	2.7301E−06	5.8585E−11	4.3287E−06	2.2156E−06	4.543E−06	3.5012E−06
BFO								
AE	1.429E−05	1.983E−05	1.735E−04	2.7634E−11	−8.3796E+02	2.59E−02	6.324E−06	9.975E−01
SD	9.603E−06	1.986E−05	2.134E−04	7.864E−11	1.304E−08	3.06E−02	5.34E−06	8.142E−01
HS								
AE	6.017E−12	2.4000E−06	2.5648E−06	6.5685E−11	−8.3796E+02	1.5391E−09	6.306E−06	6.9944E−14
SD	9.574E−12	1.708E−06	4.730E−06	7.864E−11	2.304E−12	2.080E−09	4.542E−06	3.501E−14
DE								
AE	1.754E−13	4.136E−07	2.869E−06	0	−8.3796E+02	3.10E−12	5.637E−12	9.42E−03
SD	3.318E−13	3.132E−07	3.86E−06	2.31E−12	1.82E−14	6.56E−12	3.72E−12	1.33E−02
ABC								
AE	4.30E−17	0	2.325E−06	0	−8.3796E+02	4.37E−17	9.64E−17	4.04E−17
SD	1.07E−17	1.34E−17	2.56E−05	1.23E−13	0	1.10E−17	5.24E−17	1.12E−17
DBC								
AE	**0**	**0**	**0**	**0**	**−8.3796E+02**	**0**	**−8.818E−17**	**0**
SD	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Case study-I: normal benchmark problems

- **DBC performed either better or at same level with the other algorithms at lower dimensions.**
- **The t-tests show that DBC perform better than most of algorithms and in a statistically significant manner.**
- **The uniqueness of the solution by the proposed algorithm is evident from the zero standard deviation (SD) which is novel character of proposed algorithm and can be recommended for real time applications.**

The *t*-test results of comparing DBC with the other algorithms.

t-Test for DBC vs.	Sphere	Schwefel's 2.22	Rosenbrock	Rotated-hyper	Schwefel's 2.26	Rastrigin	Ackley	Griewank
GA	1	1	1	1	0	1	1	1
EA	1	1	1	0	0	1	1	1
PSO	1	1	1	1	0	1	1	1
BFO	1	1	1	1	0	1	1	1
HS	1	1	1	1	0	1	1	1
DE	1	1	1	0	0	1	1	1
ABC	1	0	1	1	0	1	1	1
DBC	1	0	1	0	0	1	1	1

Case study-II: scalability study

- **The dimension of the functions is increased from 2 to 30 and the performance with different algorithms is noted.**

Mean and standard deviation (SD) of the benchmark function optimization results ($N = 30$).

Algorithm	Functions							
	Sphere	Schwefel's 2.22	Rosenbrock	Rotated-hyper	Schwefel's 2.26	Rastrigin	Ackley	Griewank
GA								
AE	1.255E−02	8.058E−01	1.6628E+02	4.8340E+00	−8.914E+03	1.0438E+01	1.099E+00	1.2342E+00
SD	4.3E−03	6.52E−01	5.951E+01	2.5505E+00	7.25E+02	2.6386E+00	2.496E−01	1.1045E−01
EA								
AE	1.789E−03	1.7207E−02	3.1318E+01	1.5891E−02	−1.166E+04	7.1789E−01	1.047E−02	4.6366E−03
SD	2.77E−04	1.70E−03	1.74E+01	4.29E−03	2.34E+00	9.22E−01	9.08E−04	3.96E−03
PSO								
AE	1.115E−02	1.0612E+00	4.0254E+02	4.412E+03	−1.0652E+04	3.2476E+00	1.492E−06	1.1151E−02
SD	1.421E−02	5.853E−01	6.336E+02	1.511E+03	6.63E+02	6.9521E+00	1.861E−06	1.4209E−02
BFO								
AE	4.143E−01	3.1097E+00	1.265E+01	3.562E−02	−1.1631E+04	1.1592E+01	1.991E+01	9.147E−03
SD	1.288E−01	1.908E−01	1.46E−01	2.54E−01	1.416E+01	2.2250E+01	3.796E−01	7.62E−03
HS								
AE	1.87E−04	1.71524E−01	3.4029E+02	4.297E+03	1.342E−02	7.157E−02	7.971E−00	1.2079E−02
SD	3.2E−05	7.2851E−02	2.6669E+02	1.362E+03	−1.2539E+04	1.3906E+00	7.450E+00	1.9128E−02
DE								
AE	8.2E−14	1.5E−09	3.246E−01	6.8E−11	−1.108E+04	6.92E+00	9.7E−08	0
SD	5.9E−14	9.9E−10	2.98E−01	7.4E−11	5.74E+02	3.88E+01	4.2E−08	0
ABC								
AE	3.018E−15	1.2168E−09	2.048E−01	5.312E−13	−1.2534E+04	3.912E−13	8.706E−10	3.979E−16
SD	2.194E−16	7.8E−11	2.914E−01	4.31E−12	1.042E+01	7.876E−13	6.52E−10	3.54E−02
DBC								
AE	**2.423E−26**	**5.3291E−14**	**2.6347E−03**	**7.6377E−24**	**−1.2567E+04**	**0**	**2.753E−14**	**0**
SD	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Case study-III: shifted and shifted rotated benchmark problems

- **A set of CEC 2005 benchmark problems shifted, shifted rotated and hybrid composite is evaluated for DBC algorithm and compared with the counterparts.**

Mean and standard deviation (SD) for shifted and shifted rotated benchmark functions (from f9 to f16).

Algorithm	Functions							
	Shifted sphere	Shifted Schwefel's 1.2	Shifted Rosenbrock	Shifted Rastrigin	Shifted rotated high conditional elliptic function	Shifted rotated Griewank	Rotated hybrid composition function CF1	Rotated hybrid composition function CF2
GA								
AE	9.75E−04	1.68	31.56	5.61E−04	1.06E+06	6.93E−01	144.63	997.37
SD	6.17E−04	1.25	33.95	3.8E−04	5.98E+05	4.31E−01	20.38	108.79
EA								
AE	1.74E+03	2.61E+03	9.13E+03	3.67E+01	6.84E+03	5.50E+03	213	897.12
SD	1.02E+02	1.36E+02	3.63E+03	4.17E+02	2.64E+02	5.55E+03	18.76	189.04
PSO								
AE	0	0	39.89	5.26	5.64E+04	4.36E−02	103.51	637.46
SD	0	0	113.23	2.74	3.27E+04	1.96E−02	8.77	265.94
EPSO								
AE	5.5775E−04	1.0545E−02	2.0754E+02	3.1543E+01	7.7264E+05	1.6035E+00	1.3963E+02	1.3167E+01
SD	8.9835E−04	1.1203E−02	7.2116E+02	8.0409E+00	1.0179E+06	9.8414E−01	4.4296E+01	7.0216E+01
BFO								
AE	9.82E−01	3.62	703.97	15.62	1.07E+05	845.67	165.04	843.1
SD	2.44E−01	8.6E−01	1.75E+03	2.04	6.92E+04	156.15	52.21	109.1
DE								
AE	0	7.53E−09	3.76E−06	0	3.21E−01	2.01E−01	109.77	407.51
SD	0	8.73E−09	6.16E−06	0	5.35E−01	1.83E−01	7.11	222.01
DDE								
AE	6.84E−16	1.89E−08	0	17.5	1.46E−15	2.89E−03	−1.03	0.398E+01
SD	5.72E−16	1.00E−08	0	6.09E+00	2.46E−15	8.48E−04	1.30E−14	9.65E−15
LA-DE								
AE	5.684E−14	3.411E−13	1.136E−13	3.979E+00	5.339E+02	3.299E−01	1.183E+02	8.470E+02
SD	4.198E−14	8.311E−12	1.211E−20	6.298E+00	2.489E+00	2.488E−01	6.886E+01	2.737E+02
HS								
AE	6.440E+00	3.888E+03	3.401E+03	8.710E−01	1.500E+07	4.080E+03	3.448E+02	7.739E+03
SD	2.777E+00	1.115E+03	3.272E+03	8.086E−01	4.456E+06	5.733E+00	2.581E+02	4.192E+03
EHS								
AE	4.343E−11	1.667E−09	4.183E−01	1.818E−06	5.024E+01	1.007E−05	1.103E+02	4.3595E+02
SD	3.866E−12	2.943E−10	1.7360E−02	5.927E−07	8.008E+01	2.064E−06	2.029E+00	7.261E+01
ABC								
AE	0	3.73	1.46	0	6.41E+05	2.45E−01	145.2	554.08
SD	0	2.52	1.67	0	2.78E+05	2.67E−01	14.7	86.66
DBC								
AE	0	0	0	0	3.16E−17	3.96E−11	1.024E+02	4.0738E+02
SD	0	0	0	0	0	0	0	0

Conclusion

- **The nature inspired algorithms are analyzed and problems are identified there by making a way for the proposal of new DBC optimization.**
- ***Upshot of the proposed algorithm:* Generates better optimal solutions as compared to its counterparts.**
- **One has to choose between consensus and quorum, can go for consensus for better accuracy.**
- **In case of quorum better solution can be obtained by increasing the threshold.**
- **The results show that DBC performs better and also *removes the randomness in the algorithm* when compared to other algorithms at lower and higher dimensions *by generating unique solution*.**
- **Other classes of improved, mutated, directed and hybrid algorithms like CLPSO, DM-GA , HSDM, EPSDE are still to be compared and will be agenda for future research.**