Linear Programming Problems

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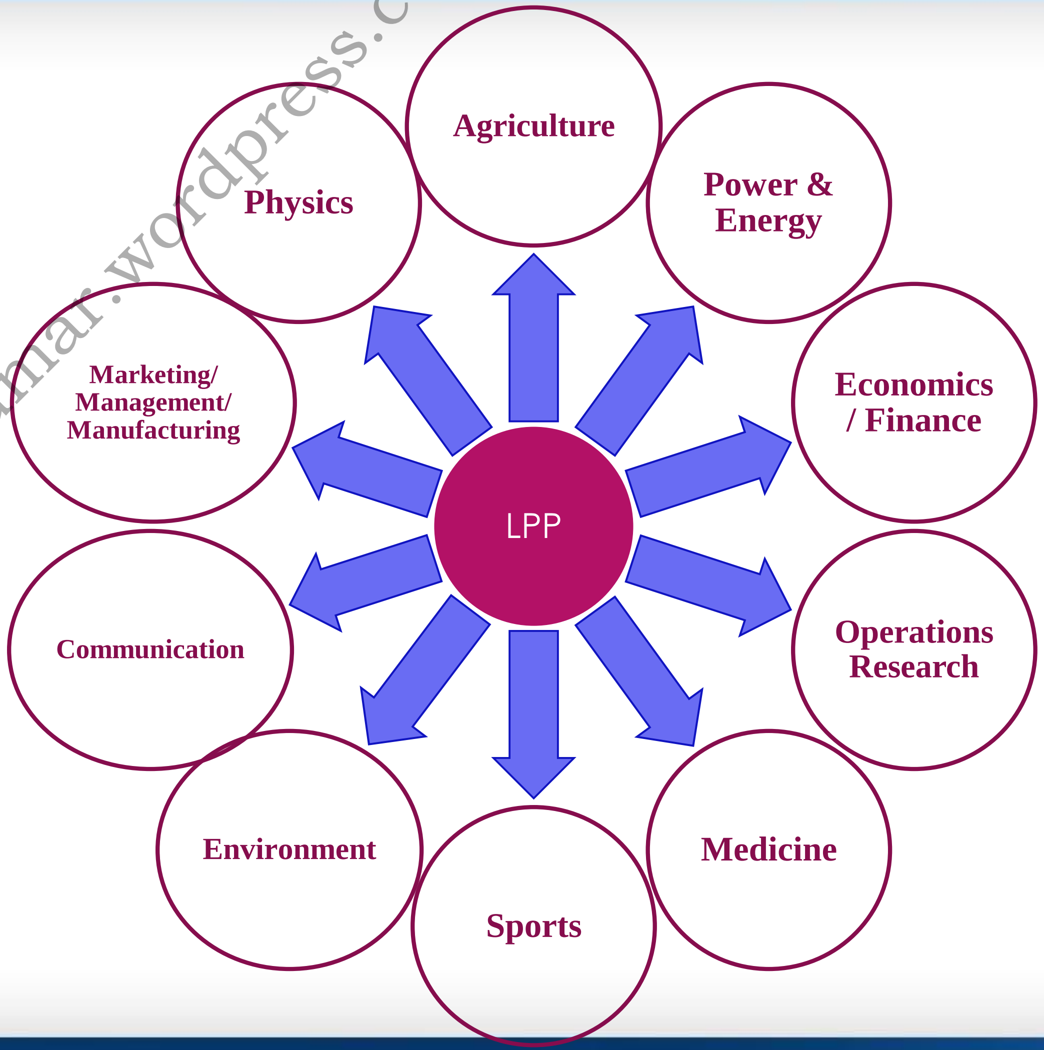
Linear Programming Problem (LPP) Applications

- Applications of Linear Programming can be seen in different fields.

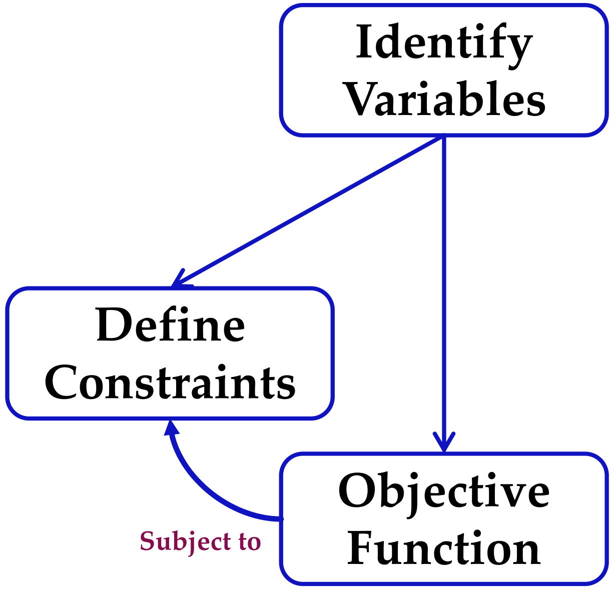
- Solution to LP for a specific problem gives solution to the problem



LPP - Formulations

- The LP is formulated by the following steps:

1. Identify the **Decision Variables** → Variables that determine the outcomes
2. Define **Constraints** → **Linear inequalities** or **equations** that add ***restrictions*** on the variables.
3. Define **Objective Function** → Based on the variables and the aim of the problem, determine a **linear function** that is to be **maximized or minimized.**



- Given that
  - Variables are **non-negative**
  - **Equations/ inequalities** are **linear** in nature.
  - All relations used are **linear relations**

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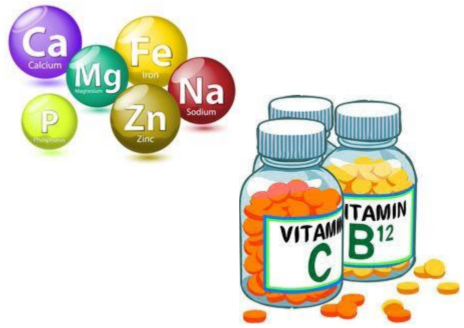
- *Diet Planning*
- *Loading of Cargo Plane*
- *Scheduling Work shifts of employees*
- *Planning of Production Amounts*
- *Airline Crew Scheduling*
- *Shortest Path Problem*
- *Energy Distribution*
- *Installation/ Placement*

Diet Planning

Diet Problem

- A dietician has to develop a special diet using **two foods P and Q** with the specifications as given in the Table.
- The diet requires **at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol**.



Goal:

How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

	Weight (g)	Ca	Fe	Cholesterol	Vitamin A
P	30	12	4	6	6
Q		3	20	4	3

Diet Problem : Formulation

Variables:

$x \rightarrow$ Packets of P
$y \rightarrow$ Packets of Q

Objective:

$$\text{minimize } Z = 6x + 3y$$

Subject to constraints (1) – (4)

Constraints:

- Equations
  - Calcium → $12x + 3y$
  - Iron → $4x + 20y$
  - Cholesterol → $6x + 4y$
  - **Vitamin A** → $6x + 3y$
- Constraint on Calcium in diet – minimum **240 units**

  $12x + 3y \geq 240 \quad \rightarrow \quad 4x + y \geq 80 \qquad (1)$
- Constraint on Iron – minimum **460 units**

  $4x + 20y \geq 460 \quad \rightarrow \quad x + 5y \geq 115 \qquad (2)$
- Constraint on Cholesterol – maximum **300 units**

  $6x + 4y \leq 300 \quad \rightarrow \quad 3x + 2y \leq 150 \qquad (3)$
- $x \geq 0$ and $y \geq 0 \qquad (4)$

Loading of Cargo Plane

Cargo Plane

- Cargo Plane has 3 compartments for storing cargo with limits on weight and space:

Compartment	Weight cap (tonnes)	Space cap (m^3)
Front (F)	10	6800
Centre (C)	16	8700
Rear (R)	8	5300

- There are four cargos available for shipment on the next flight.

Cargo	Weight (tonnes)	Volume (m^3/tonne)	Profit (pd/tonne)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

- The **weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane.**
- Any proportions of C1-C4 may be accepted.

Goal:

i) Determine the amount of each C1-C4 that should be accepted.

ii) How to distribute so that total profit of the flight is maximized?

Cargo Plane: Formulation

Variables:

i → cargo (C1 → C4 as 1→ 4)
j → compartment (1 = F, 2 =C, 3=R)
x_{ij} → number of tonnes of cargo

Constraints:

- Constraint on maximum available cargo

$$x_{11} + x_{12} + x_{13} \leq 18$$
$$x_{21} + x_{22} + x_{23} \leq 15$$
$$x_{31} + x_{32} + x_{33} \leq 23$$
$$x_{41} + x_{42} + x_{43} \leq 12$$

Constraints:

- Constraint weight capacity of each compartment

$$x_{11} + x_{21} + x_{31} + x_{41} \leq 10$$
$$x_{12} + x_{22} + x_{32} + x_{42} \leq 16$$
$$x_{13} + x_{23} + x_{33} + x_{43} \leq 8$$

- Constraint on space capacity of each compartment

$$480x_{11} + 650x_{21} + 580x_{31} + 390x_{41} \leq 6800$$
$$480x_{12} + 650x_{22} + 580x_{32} + 390x_{42} \leq 8700$$
$$480x_{13} + 650x_{23} + 580x_{33} + 390x_{43} \leq 5300$$

- Weight of cargo in compartments = proportion of the compartment's weight capacity

$$(x_{11} + x_{21} + x_{31} + x_{41})/10 = (x_{11} + x_{22} + x_{32} + x_{42})/16$$
$$= (x_{13} + x_{23} + x_{33} + x_{43})/8$$

Cargo Plane: Formulation

Variables:

- i → cargo (C1 → C4 as 1→ 4)
- j → compartment (1 = F, 2 =C, 3=R)
- x_{ij} → number of tonnes of cargo

Constraint 1: Cargo

$$x_{11} + x_{12} + x_{13} \leq 18$$
$$x_{21} + x_{22} + x_{23} \leq 15$$
$$x_{31} + x_{32} + x_{33} \leq 23$$
$$x_{41} + x_{42} + x_{43} \leq 12$$

Constraint 2: Weight capacity

$$x_{11} + x_{21} + x_{31} + x_{41} \leq 10$$
$$x_{12} + x_{22} + x_{32} + x_{42} \leq 16$$
$$x_{13} + x_{23} + x_{33} + x_{43} \leq 8$$

Constraint 3: Space Capacity

$$480x_{11} + 650x_{21} + 580x_{31} + 390x_{41} \leq 6800$$
$$480x_{12} + 650x_{22} + 580x_{32} + 390x_{42} \leq 8700$$
$$480x_{13} + 650x_{23} + 580x_{33} + 390x_{43} \leq 5300$$

Constraint 4: Weight Proportions

$$(x_{11} + x_{21} + x_{31} + x_{41})/10 = (x_{11} + x_{22} + x_{32} + x_{42})/16$$
$$= (x_{13} + x_{23} + x_{33} + x_{43})/8$$

↓

Objective:

$$\textit{maximize } Z = 310(x_{11} + x_{12} + x_{13}) + 80(x_{21} + x_{22} + x_{23}) + 350(x_{31} + x_{32} + x_{33}) + 285(x_{41} + x_{42} + x_{43})$$

Subject to constraints (1) – (4).

Scheduling Work Shifts for Employees

Work Shift Scheduling

- Production manager is attempting to **develop a shift pattern** for the workforce.
- Each day is divided into **3 eight hour shift periods 00:01 – 08:00 (Night), 08:01 – 16:00 (Day); 16:01 – 24:00 (Late)**
- Minimum number of workers required for each of these shifts over any week is in Table.
  - Each worker is assigned to work either a night shift or a day shift or a late shift.
  - Once a worker has been assigned to a shift, they must **remain on the same shift every day that they work**
  - Each worker **works 4 consecutive days** during any seven day period.
- **Total number of workers = 60**

Goal:
Develop a schedule so that he workforce size is reduced for he week.

	Mon (M)	Tue (T)	Wed (W)	Thurs (Th)	Fri (F)	Sat (Sa)	Sun (S)
Night (N)	5	3	2	4	3	2	2
Day (D)	7	8	9	5	7	2	5
Late (L)	9	10	10	7	11	2	2

Work Shift Scheduling : Formulation

Variables:

$i \rightarrow$ Days: $1 \rightarrow 7$ (1 = M, 2 = T,….7 = S)
$j \rightarrow$ Shift: $1 \rightarrow 3$ (1 = N, 2 = D, 3 = L)

- $N_{ij} \rightarrow$ Number of workers starting their four consecutive workdays on day *i* and shift *j*
- $D_{ij} \rightarrow$ *Known* number of workers required on day *i* and shift *j*

If a worker is at a shift j on Day 1 then

- → *their work started on Day 1 (N_{1j}) or Day 7 (N_{7j}), Day 6(N_{6j}) or Day 5 (N_{5j}).*
- → *Sum of these variables gives the total number of workers on the day and in j shift!*

Constraints:

- Constraint on total number of workers **60**

$$\sum_{i=1}^{7}\sum_{j=1}^{3} N_{ij} \le 60$$

- Constraint on minimum number of workers required per day per shift + constraint of working 4 consecutive days in a 7 day period.

$$M \rightarrow N_{1j} + N_{7j} + N_{6j} + N_{5j} \ge D_{1j} \quad j = 1, \ldots, 3$$

$$T \rightarrow N_{2j} + N_{1j} + N_{7j} + N_{6j} \ge D_{2j} \quad j = 1, \ldots, 3$$

$$W \rightarrow N_{3j} + N_{2j} + N_{1j} + N_{7j} \ge D_{3j} \quad j = 1, \ldots, 3$$

.

$$S \rightarrow N_{7j} + N_{6j} + N_{5j} + N_{4j} \ge D_{7j} \quad j = 1, \ldots, 3$$

Work Shift Scheduling : Formulation

Variables:

i → Days: 1→ 7 (1 = M, 2 = T,….7 = S)
j → Shift: 1→3 (1 = N, 2 = D, 3 = L)

- N_{ij} → Number of workers starting their four consecutive workdays on day *i and shift j*
- D_{ij} → *Known* number of workers required on day *i* and shift *j*

Objective (Subject to constraints):

$$minimize\ Z = \sum_{i=1}^{7}\sum_{j=1}^{3} N_{ij}$$

Constraints:

- Constraint on total number of workers 60

$$\sum_{i=1}^{7}\sum_{j=1}^{3} N_{ij} \leq 60$$

- Constraint on minimum number of workers required per day per shift + constraint of working 4 consecutive days in a 7 day period.

M → $N_{1j} + N_{7j} + N_{6j} + N_{5j} \geq D_{1j}$ j = 1, …,3

T → $N_{2j} + N_{1j} + N_{7j} + N_{6j} \geq D_{2j}$ j = 1, …,3
W → $N_{3j} + N_{2j} + N_{1j} + N_{7j} \geq D_{3j}$ j = 1, …,3 .
S → $N_{7j} + N_{6j} + N_{5j} + N_{4j} \geq D_{7j}$ j = 1, …,3

Planning for Production Amount

Production Planning Problem

- 3 stages of assembly with respective man-hours required are shown in the table.
- Official time for each stage → **160, 200 and 80** man hours.

Product	Man hours in each stage			Profit ($)	Max Demand (units)
	A	B	C		
P1	2	2	3	10	50
P2	2	4	6	15	60
P3	1	1	1	22	85
P4	1	2	5	17	70

Given

- If worker is previously employed on **stage B → can spend up to 20% of their time on Stage A**
- If worker is previously employed on **stage C → can spend up to 30% of their time on Stage A**
- Ratio of **(P1 units assembled)/(P4 units assembled) = 0.9 → 1.15**

Goal:

Decide how much product is to be assembled for the next week.

Production Planning Problem: Formulation

Variables:

$i \rightarrow$ **Product {I = 1,2,3,4}**

$x_i \rightarrow$ **amount of Product i produced**

$t_{BA} \rightarrow$ **amount of time transferred B → A**

$t_{CA} \rightarrow$ **amount of time transferred C → A**

Objective :

$$\text{maximize } Z = 10x_1 + 15x_2 + 22x_3 + 17x_4$$

Subject to constraints

Constraints:

- Constraint on maximum demand

  $x_1 \le 50;\ x_2 \le 60;\ x_3 \le 85;\ x_4 \le 70$

- Constraint on P1/P4 ratio

  $0.9x_4 \le (x_1/x_4) \le 1.15x_4$

- Constraint on work time

  $2x_1 + 2x_2 + x_3 + x_4 \le 160 + t_{BA} + t_{CA}$

  $2x_1 + 4x_2 + x_3 + 2x_4 \le 200 - t_{BA}$

  $3x_1 + 6x_2 + x_3 + 5x_4 \le 80 - t_{CA}$

- Constraint on transferred time

  $t_{BA} \le 0.2(200)$

  $t_{CA} \le 0.3(80)$

- All variables are ≥ 0

Airline Crew Scheduling

Airplane Crew Scheduling

- All airlines (*Air India, IndiGo, Emirates, British Airways*) have a large number of flights travelling all over the world.
  - **Large number of sources/ destination airports.**
  - Large number of **flights outgoing** from a single node (airport) at **different times of the day**
  - Duration of each trip is different.

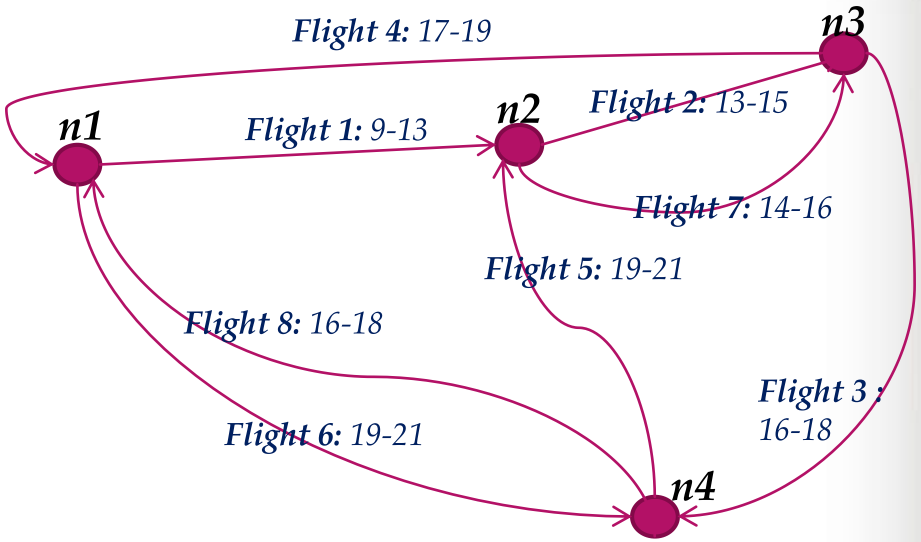
Necessary to assign sufficient number of crew members to each flight.

- Regulations and limitations have to be taken into consideration
  - **Pilots have a certain number of flying hours/ day**
  - Crew members also have maximum travel limits in *t* time period.
  - If **crew member has a layover, accommodation is arranged by airlines**.

Difficult to organize!!!!!

Airplane Crew Scheduling

- Consider an airline that has to **assign *m* crew** members to each flight flying **to/from *n* nodes**.
- Given the directed graph showing 4 nodes between which the flights are travelling [1][2]
- Given:
  - Each crew **must service at least 2 flights** → Each pairing must contain 2 flights
  - **Multiple crews may be assigned to a flight** (to transport a crew to another airport)
  - Each flight must have at least 1 crew.
- The **cost of a pairing** is expressed as the time interval between the first departure time and the last arrival time as **+5 hours**



Goal:
Minimize the cost function while assigning crews to the flights

Airplane Crew Scheduling: Formulation

Variables:

n → Flights $1 \le j \le n$
m → crew $1 \le i \le m$

- **Decision Variable →** y_{ij}

$$y_{ij} = \begin{cases} 1 & \text{flight } j \text{ has crew } i \\ 0 & \text{otherwise} \end{cases}$$

Constraints:

- Each flight (j) must have at least 1 crew

$$\sum_{i=1}^{n} y_{ij} \ge 1$$

- **Can form *pairings of flights* that are in sequence so that crew may have time to service two (or more) flights**

Airplane Crew Scheduling: Formulation

- Tabulated Departures/Arrivals
- Look at flight pairings
  - Crew may go to next flight as long as the flight arrives at or before the departure
  - Minimum flight that each must serve is 2
- Total pairings ➔ 21

Flight	S	D	Departure from S	Arrival at D
1	N1	N2	900hrs (9:00am)	1300hrs (9:00am)
2	N2	N3	1300hrs (1:00pm)	1500hrs (3:00pm)
3	N3	N4	1600hrs (4:00pm)	1800hrs (6:00pm)
4	N3	N1	1700hrs (5:00pm)	1900hrs (7:00pm)
5	N4	N2	1900hrs (7:00pm)	2100hrs (9:00pm)
6	N4	N1	1900hrs (7:00pm)	2100hrs (9:00pm)
7	N2	N3	1400hrs (2:00pm)	1600hrs (4:00pm)
8	N1	N4	1600hrs (4:00pm)	1800hrs (6:00pm)

x_i	Pairings	Total time (t)
1	1→2	6(4+2)
2	1→2→3	9(4+2+1+2)
3	1→2→4	10
4	1→7→3	9
5	1→7→4	10
6	1→7→3→5	12
7	1→7→3→6	12
8	1→2→3→5	12
9	1→2→3→6	12
10	2→3	5

x_i	Pairings	Total time (t)
11	2→ 4	6
12	7→3	4
13	7→4	6
14	7→3→5	7
15	7→3→6	7
16	2→3→5	8
17	2→3→6	8
18	8→6	5
19	3→5	5
20	3→6	5
21	1→7	7

$\underbrace{\hspace{3cm}}_{\textit{All possible parings}}$

Airplane Crew Scheduling: Formulation

Variables:

n → Flights $1 \le j \le n$
m → crew $1 \le i \le m$

- **Decision Variable** → y_{ij}

$$y_{ij} = \begin{cases} 1 & \text{flight } j \text{ has crew } i \\ 0 & \text{otherwise} \end{cases}$$

- *Variable for each pairing*

$$x_i = \begin{cases} 1 & \text{pairing } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Constraints:

- Each flight (j) must have at least 1 crew

$$\sum_{i=1}^{n} y_{ij} \ge 1$$

- **Inequalities based on pairings**

- **For each flight, inequalities exist with respect to each pairing.**
- **The inequality is formed by adding each x_i the flight number is in!**

Airplane Crew Scheduling: Formulation

Variables:

n → Flights $1 \le j \le n$
m → crew $1 \le i \le m$

- Decision Variable → y_{ij}

$$y_{ij} = \begin{cases} 1 & \text{flight } j \text{ has crew } i \\ 0 & \text{otherwise} \end{cases}$$

- *Variable for each pairing*

$$x_i = \begin{cases} 1 & \text{pairing } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Constraints:

- Each flight (j) must have at least 1 crew

$$\sum_{i=1}^{n} y_{ij} \ge 1$$

- **Inequalities based on pairings**

Flight 1 → $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{17} \ge 1$
Flight 2 → $x_1 + x_2 + x_3 + x_8 + x_9 + x_{10} + x_{11} + x_{16} + \ge 1$
.
Flight 8 → $x_{18} \ge 1$

- **Equations for each** $x_i \rightarrow (t_i + 5)x_i$

Objective:

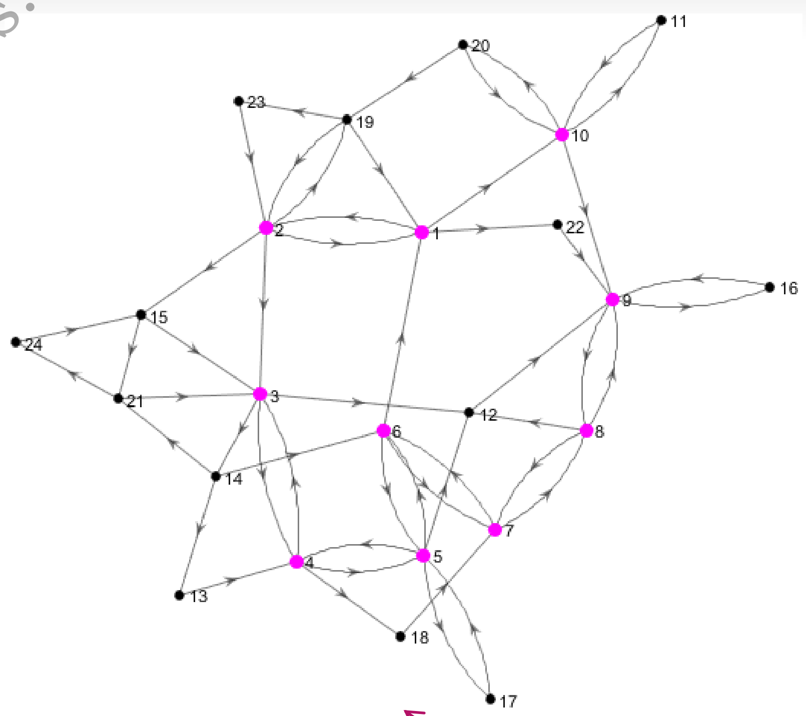
$$\text{minimize } Z = 11x_1 + 14x_2 + 15x_3 + \ldots\ldots. + 12x_{21}$$

Subject to constraints

Searching for Shortest Path between Nodes

Shortest Path Problem

- Aim → Find the shortest path between source node(s) and destination node(s)
- Requires a weighted graph/digraph (graph with directed edges connecting nodes)



Digraph - Directed graph

Weight of path p = $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_k$ is

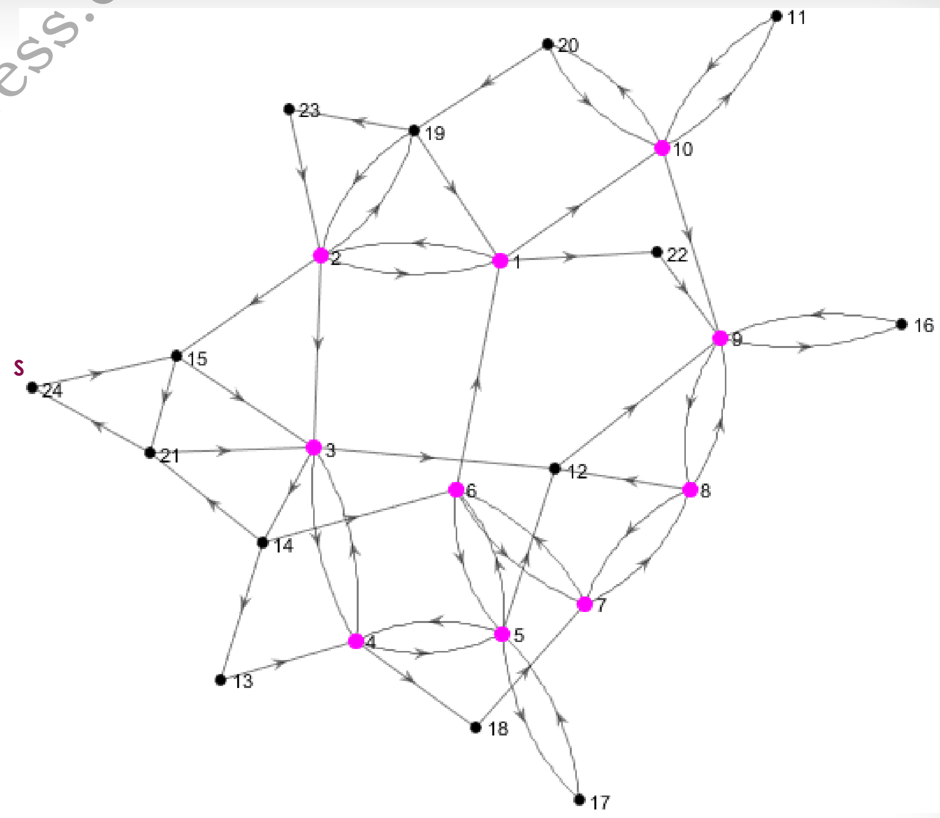
$$w(p) = \sum_{i=1}^{k-1} w(v_i, v_{i+1})$$

Variants of Shortest Path

- Shortest-Path problems
  - Single-source shortest-paths problem: Find the shortest path from *s* to each node.
  - Single-destination shortest-paths problem: Find a shortest path to a given *destination* target *t* from each node.
  - Single-pair shortest-path problem: Find a shortest path from $i \rightarrow j$ for given vertices *i* and *j*.
  - All-pairs shortest-paths problem: Find a shortest path from $i \rightarrow j$ for every pair of vertices *i* and *j*.

Shortest Path Problem: Single source & destination

- Given a directed graph, with **source node 's ='** and **target (destination) node 't'** .
- The **cost is the weight (w_{ij})** of the edge connecting a node i to node j.
- Shortest Path Problem deals with finding the shortest path between s and t.
  - ***Path with the least weight***



Goal:
Minimize the cost function by finding the shortest path between source and target nodes.

Shortest Path as Linear Programming: Formulation

Variables:

- s → *source node*
  t → *target node*
  i → *start node for sub-path*
  j → *target node for sub-path*
  s → *source node*
- w_{ij} → *Edge weight between node i and j*
- $x_{ij} = \begin{cases} 1 & \text{edge(i,j) is part of shortest path} \\ 0 & \text{otherwise} \end{cases}$

Constraints:

- Constraints

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}$$

Objective (Subject to constraints):

$$\text{minimize } Z = \sum_{ij \in A} w_{ij} x_{ij}$$

Energy Distribution

Energy Distribution

- A company distributing electric energy has several power and distributing stations connected by wires.
- Each station i can:
  - produce p_i kW of energy ($p_i = 0$ if the station cannot produce energy)
  - distribute energy on a sub-network whose users have a total demand of d_i kW ($d_i = 0$ if the station serve no users)
  - carry energy from/to different stations.
- The wires connecting station i to station j
  - Maximum capacity of u_{ij} kW
  - Cost of c_{ij} dollars for each kW carried by the wires.
- Given that the total amount of energy produced equals the total amount of energy required by all sub-networks.

Goal

The distribution company wants to determine the minimum cost distribution plan.

Energy Distribution: Formulation

- Consider a digraph (directed graph) G = (N,A)
  - Nodes represent power/distribution stations
  - Arcs (edges) represent connections or wires between pairs of stations.
- Define a variable $b_v = d_v - p_v$ → difference between energy demand (d_v) and energy available (p_v) at a station (node).
- Define nodes as
  - Demand nodes: $b_v > 0$ → $d_v > p_v$.
  - Supply nodes: $b_v < 0$ → $d_v < p_v$.
  - Transshipment nodes: $b_v = 0$ → $d_v = p_v$, i.e.this node will serve as an intermediate node between stations.
- **Node balancing** → difference between the total flow (of electric energy) entering node v (edge (i, v)) and the total flow leaving node v (edge (v, j)) is exactly equal to the demand of the node

Energy Distribution: Formulation

Variables:

- p_i → *Energy produced at node i*
- d_i → *Demand at node i*
- x_{ij} → *energy that should be available on edge (i,j)*
- u_{ij} → *maximum capacity of edge(i,j)*
- c_{ij} → *cost/kW on edge(i,j)*

Constraints:

- **Node balance constraint**

$$\sum_{(i,j)\epsilon A} x_{iv} - \sum_{(v,j)\epsilon A} x_{vj} = bv \qquad \forall v \epsilon N$$

- **Arc (Edge) Capacity constraint**

$$x_{ij} \leq uij \qquad \forall (i,j)\epsilon A$$

Objective:

$$minimize\ Z = \sum_{(i,j)\epsilon A} c_{ij}x_{ij}$$

Subject to constraints

Determining Installation/Placement site(s)

Antenna location sites

- **5 possible sites** for installation of antennas have been detected by a telephone company, in order to **cover 6 areas**.
- **Intensity of signal** coming from each antenna at each site (A - E) is given through simulation (in Table).
- **Receivers** recognize only **signals whose level** is at least **18**.
- **More than one signal may not reach level 18 in the same area** → Would cause an interference
- An antenna may be **placed in site E only if an antenna is installed in site D.**

	AREA 1	AREA 2	AREA 3	AREA 4	AREA 5	AREA 6
SITE A	10	20	16	25	0	10
SITE B	0	12	18	23	11	6
SITE C	21	8	5	6	23	19
SITE D	16	15	15	8	14	18
SITE E	21	13	13	17	18	22

Goal:

Placement of antennas to ensure coverage over maximum number of areas.

Antenna location sites: Formulation

Variables:

- $i \rightarrow$ set of sites locations (i = 1,2….5)
- $j \rightarrow$ set of areas (j = 1,2,….,6)
- $T \rightarrow$ minimum signal level
- $N \rightarrow$ max number of signals above the threshold recognized by receiver ($N = 1$)
- $s_{ij} \rightarrow$ signal strength of antenna at i received in j
- $M_j \rightarrow$ sufficiently large parameter, so that for every area $j \in J$, $M_j = card(\{i \in I : s_{ij} \geq T\})$
- $x_i = \begin{cases} 1 & \textit{antenna placed at i} \\ 0 & \textit{otherwise} \end{cases}$
- $z_j = \begin{cases} 1 & \textit{area j is covered} \\ 0 & \textit{otherwise} \end{cases}$

Constraints:

- Minimum signal level

$$s_{ij} \geq T$$

- Number of antenna covering an area

$$\sum_{i\epsilon I} x_i \geq zj$$

- $x_i \epsilon \{0,1\}$ and $z_j \epsilon \{0,1\}$

Objective :

$$\text{maximize} \sum_{j\epsilon J} z_j$$

Subject to constraints

Thank You!

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Other Problems: Distributing Different Kinds of Energy

Distributing different kinds of energy

- A company distributing electric energy has several power and distributing stations connected by wires. Each station produces/distributes different kinds of energy. Each station i can: • produce p k i kW of energy of type k (p k i = 0 if the station does not produce energy of type k); • distribute energy of type k on a sub-network whose users have a total demand of d k i kW (d k i = 0 if the station serve no users requiring energy of type k); • carry energy from/to different stations. Note that every station can produce and/or distribute different types of energy. The wires connecting station i to station j have a maximum capacity of uij kW, independently of the type of energy carried. The transportation cost depends both on the pair of stations (i, j) and the type of energy k, and is equal to c k ij euros for each kW. The company wants to determine the minimum cost distribution plan, under the assumption that, for each type of energy, the total amount produced equals the total amount of energy of the same type required by all sub-networks.

Other Problems: Communication Network 2

Communication Network 2

- A communication network consists of routers and connections routes between pairs of routers. Every router generates traffic towards every other router and, for every (ordered) pair of routers, the traffic demand has been estimated (this demand is measured in terms of bandwidth required). The traffic from router i to router j uses multi-hop technology (the traffic is allowed to go through intermediate nodes) and splittable flow technology (the traffic can be split along different paths). For every route, the capacity (how much flow can be sent) is known, and the unit cost for each unit of flow is also known. The target is to send the data flow at the minimum cost.